A list of formulas: $I = Prt$; $A = P(1+rt)$; $A = P(1+i)^n$; $APY = (1+\frac{r}{m})^m - 1$;

$APY = e^r - 1$; $FV = PMT\frac{(1+i)^n-1}{i}$, $PV = PMT\frac{[1-(1+i)^{-n}]}{i}$, where $i = \frac{r}{m}$ and $n = mt$

1. (10 points) What annual nominal rate compounded semiannually has the same annual percentage yield as 6.5% compounded continuously?

$$\left(1+\frac{r}{2}\right)^2 - 1 = e^{0.065} - 1$$

$$\left(1+\frac{r}{2}\right)^2 = e^{0.065}$$

$$r = \left(\sqrt{e^{0.065}} - 1\right) \times 2$$

$$r = 0.06607 \qquad 6.607\,\%$$

2. (10 points) A bicycle producing company calculates that the total cost of producing x bicycles is given by the cost function $C(x) = 10000 + 50x$. If the company sells x bicycles, the market price for each bicycle is given by the equation $price = p(x) = 100 - 0.05x$. How many bicycles should the company sell to maximize its total profit? What is the maximum profit? What is the price per bicycle when the profit is maximal?

$p(x) = 100 - 0.05x$

$R(x) = x(100 - 0.05x) = 100x - 0.05x^2$

$\text{Profit}(x) = R(x) - C(x) = 100x - 0.05x^2 - (10{,}000 + 50x)$

$\text{Profit}(x) = 50x - 0.05x^2 - 10{,}000 \qquad x \in (0, 2000)$

$p'(x) = 50 - 0.1x = 0 \Rightarrow x = 500 \rightarrow$ only critical value

$p''(x) = -0.1 \qquad p''(500) = -0.1 < 0 \Rightarrow$ abs max at $x=500$

$P(500) = 50 \times 500 - 0.05(500)^2 - 10{,}000 = 25{,}000 - 12500 - 10{,}000$

$= \boxed{2500} \rightarrow$ abs max profit

price per bicycle $= 100 - 0.05 \times 500 = \boxed{75}$

The Company should sell 500 bicycles at a price of 75 to obtain a total profit of 2500.

3. (10 points) Solve the following system of linear equations by using the Gauss Jordan elimination. Write the solution set and determine if the system is dependent, independent, consistent or inconsistent.

$$\begin{cases} x_1 + x_2 + x_3 &= 2 \\ 2x_1 + x_2 + x_3 &= 1 \\ 3x_1 + 2x_2 + 2x_3 &= 3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 1 & 1 & 1 \\ 3 & 2 & 2 & 3 \end{array}\right] \quad \begin{array}{l} -2R_1 + R_2 \to R_2 \\ -3R_1 + R_3 \to R_3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & -1 & -1 & -3 \end{array}\right] \; -R_2 \to R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & -1 & -1 & -3 \end{array}\right] \quad \begin{array}{l} -R_2 + R_1 \to R_1 \\ R_2 + R_3 \to R_3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = -1$

$x_2 + x_3 = 3$

$$SS = \{(-1, 3 - x_3, x_3) \mid x_3 \in \mathbb{R}\}$$

Consistent and dependent system

4. Evaluate the following limits

a-)(5 points) $\lim_{x \to -\infty} \frac{-x^3+1}{5} =$ $+\infty$

b-) (5 points) $\lim_{x \to 2} \frac{x^2-4}{x^2-3x+2} =$ $\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x-1)} = \lim_{x \to 2} \frac{x+2}{x-1} = 4$

c-) (5 points) $\lim_{x \to 1} \frac{x+5}{(x-1)^2} =$ $+\infty$

$\lim_{x \to 1^+} \frac{x+5}{(x-1)^2} = +\infty$

$\lim_{x \to 1^-} \frac{x+5}{(x-1)^2} = +\infty$

5. (5 points) Find m so that $f(x) = \begin{cases} x^2 - m & if\ x \leq 1 \\ -x^2 + m & if\ x > 1 \end{cases}$ is continuous for all real numbers. Show all work to get full credit.

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 - m = 1 - m$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} -x^2 + m = -1 + m$

f is continuous at $x = 1$ so $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \Rightarrow$

$1 - m = -1 + m \Rightarrow 2 = 2m \Rightarrow m = 1$

6. Find the derivatives of following functions

a-)(5 points) $f(x) = xe^x + 2x$

$f'(x) = e^x + xe^x + 2.$

b-) (5 points) $g(x) = 5 + \frac{1}{\sqrt{x}} + \ln x + \ln 2$

$g(x) = 5 + x^{-1/2} + \ln x + \ln 2$

$g'(x) = \frac{-1}{2} x^{-3/2} + \frac{1}{x} = \frac{1}{x} - \frac{1}{2\sqrt{x^3}}$

7. (5 points) Find the equation of the tangent line to the graph of $f(x) = x^3 + 3x + 5$ at $x = 1$.

$f'(x) = 3x^2 + 3$

$f(1) = 1 + 3 + 5 = 9$

$f'(1) = 3 + 3 = 6$

f passes through $(1, 9)$

Slope of the tangent line is 6.

$y = 6x + m$

$9 = 6 + m$

$m = 3$

$\boxed{y = 6x + 3}$

8. Evaluate the following integrals

(5 points) a-) $\int e^x + \frac{1}{x} + \frac{1}{x^2}\, dx$ $= \int e^x + \frac{1}{x} + x^{-2}\, dx$

$$= \boxed{e^x + \ln|x| - \frac{1}{x} + C}$$

b-) (5 points) $\int_1^4 3\sqrt{x}\, dx$ $= \int_1^4 3x^{1/2}\, dx = 3\int_1^4 x^{1/2}\, dx$

$$= \cancel{3}\, \frac{x^{3/2}}{\cancel{3}/2}\Big|_1^4 = 2\sqrt{x^3}\Big|_1^4 = 2\sqrt{4^3} - 2\sqrt{1^3}$$

$$= 16 - 2 = \boxed{14}$$

9. (5 points) Find $f(x)$ if $f'(x) = 6x^2 - 4x$ and $f(0) = 3000$.

$$\int 6x^2 - 4x\, dx = \frac{6x^3}{3} - 4\frac{x^2}{2} + C$$

$$f(x) = 2x^3 - 2x^2 + C$$

$$f(0) = C = 3000$$

$$\boxed{f(x) = 2x^3 - 2x^2 + 3000}$$

10. (10 points)A factory makes 2 products, A and B using the same production process for each. A unit of A take 1.5 hours and a unit of B takes 0.75 hours to produce. In addition, every unit B has to be hand finished, an activity taking 0.5 hours per unit. Each week total production time (excluding hand finishing) must not exceed 300 hours and hand finishing must not exceed 30 hours.

Each unit of product A is sold for 40 TL whilst each unit of B sells for 55 TL. At most 130 of A and 100 of B can be sold each week. Formulate the problem of planning weekly production to maximize total revenue as a linear programming problem in 2 variables. Write all restrictions (Do not solve the system)

	Production	Hand Finishing	Price	
A	1.5 h	0	40 TL	≤ 130
B	0,75 h	0,5 h	55 TL	≤ 100
Total Available	300 h	30 h		

Maximize $40A + 55B$

Subject to

$$1,5A + 0,75B \leq 300$$

$$0,5B \leq 30$$

$$A \leq 130$$

$$B \leq 100$$

$$A, B \geq 0$$

11. (8 points) A company has 8 women and 5 men in its administrative team.

(i) In how many ways the company can choose a general secretary, an under secretary and an archivist?

(ii) In how many ways the company can choose 3 people to work as a team together?

(iii) How many of the selections in part (*i*) are entirely women?

(iv) How many of the selections in part (*ii*) consists of 2 women and a man?

(i) $13 \times 12 \times 11 = 1716$

(ii) $\binom{13}{3} = \frac{13!}{3! \times 10!} = \frac{13 \times \cancel{12}^{\,2} \times 11}{\cancel{3} \times \cancel{2} \times 1} = 286$

(iii) $8 \times 7 \times 6 = 336$

(iv) $\binom{8}{2}\binom{5}{1} = \frac{8 \times 7}{2} \times 5 = 140$

12. (5 points) Sketch the graph of $f(x) = \log_2(x+2)$