A list of formulas: $I = Prt$; $A = P(1+rt)$

$$A = P(1+i)^n;\ APY = (1+\frac{r}{m})^m - 1$$

$$FV = PMT\frac{(1+i)^n-1}{i},\ \text{where } i = \frac{r}{m} \text{ and } n = mt$$

1.

a-) (10 points) Ali joins a retirement plan of an insurance company. If he makes deposits of \$7500 each year for 20 years into this plan which pays 8% compounded annually, how much money will Ali receive when he retires in 20 years? How much of this amount is interest?

$$PV = 7500 \times \frac{(1+0.08)^{20}-1}{0.08} = 343,214.73$$

Interest: $343,214.73 - (7500 \times 20) = 193,214.73$

b-) (10 points) How long will it take for an investment to double if it is invested at 10% compounded quarterly?

$$2P = P\left(1+\frac{0.1}{4}\right)^n$$

$$2 = (1.025)^n$$

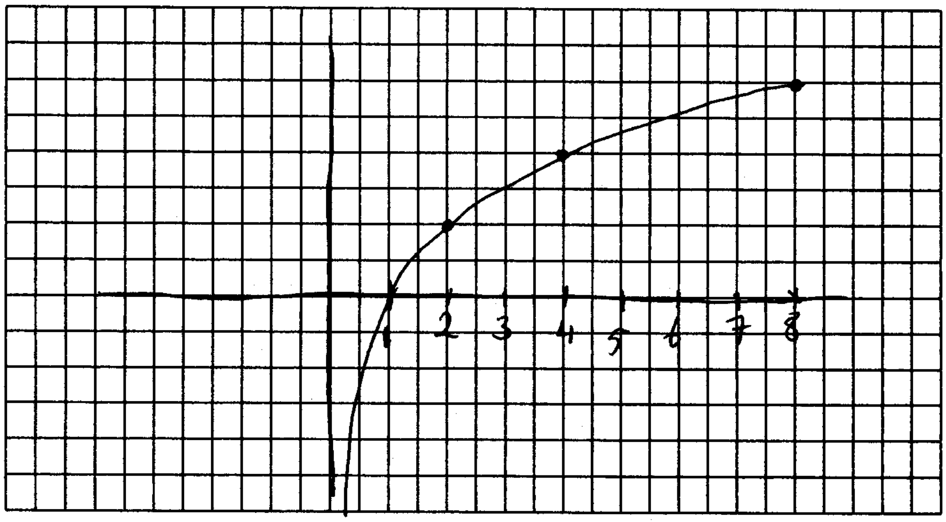
$$\ln 2 = n \ln 1.025$$

$$n = \frac{\ln 2}{\ln 1.025} \cong 28.07$$

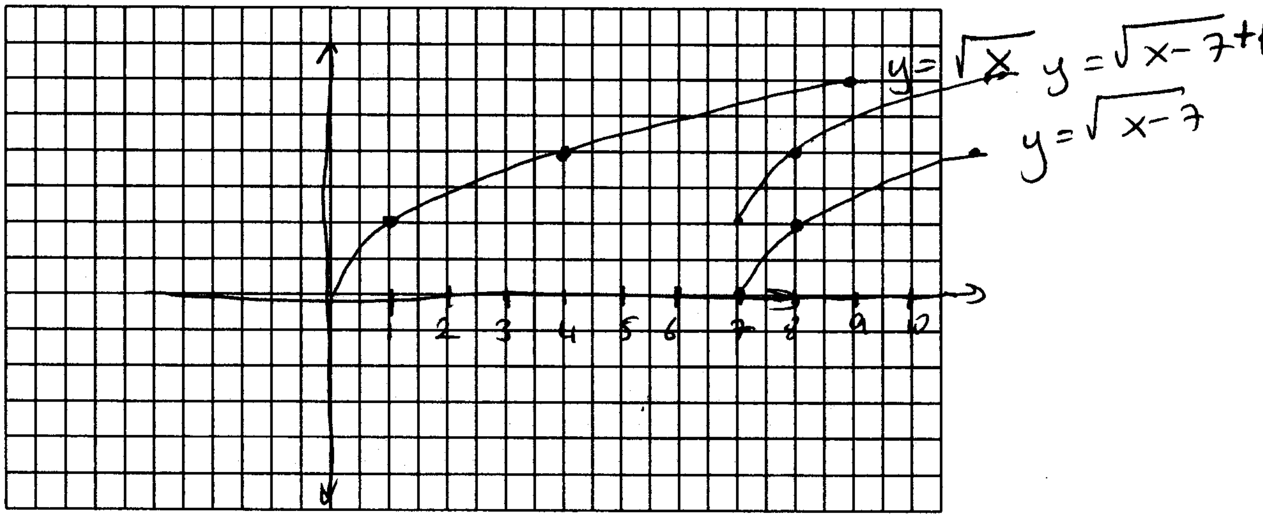
29 quaters are needed

7 years and 3 months

2. (6 points) Sketch the graph of the function $f(x) = \log_2 x$



b-)(9 points) Sketch the graph of the function $f(x) = 1 + \sqrt{x-7}$ by using horizontal translation, vertical translation, stretch and shrink where it is appropriate. Indicate each step. Specify x and y intercepts.



3. State which of the following equations define a function. If the equation defines a function find the domain and range of the function; determine if the function is one-to-one or not. If the function is one-to-one then find its inverse. Give reasons. Explain your answers.

a) (8 points) $e^y = x$

$y = \ln x$ defines a function $f(x) = \ln x$

Domain: $(0, +\infty)$

Range: $\mathbb{R}$

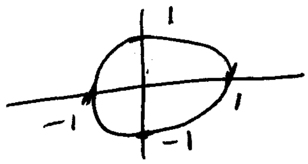
one-to-one.

$f^{-1}(x) = e^x$

b) (5 points) $x^2 + y^2 = 1$

Does not define a function

$(0,1)$ and $(0,-1)$ are both solutions to this equation



c) (7 points) $x^3 + y = x^2$

$y = x^2 - x^3$ defines a function $f(x) = x^2 - x^3$

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

This function is not 1-1 since $f(0) = f(1) = 0$

4. (15 points) Find the domain, range, intercepts and the vertex of the parabola $f(x) = -x^2 + 8x - 7$ using completing the square technique. Sketch the graph of the function.

$$f(x) = -x^2+8x-7 = -(x^2-8x)-7$$
$$= -(x^2-8x+16-16)-7$$
$$= -(x-4)^2+16-7$$
$$= -(x-4)^2+9$$

vertex of the parabola: $(4,9)$ Domain: $\mathbb{R}$

Axis of symmetry: $x=4$ Range: $(-\infty, 9]$

Intercepts: x-intercepts

$$-x^2+8x-7=0$$
$$x^2-8x+7=0$$
$$\begin{matrix} x & -7 \\ x & -1 \end{matrix}$$

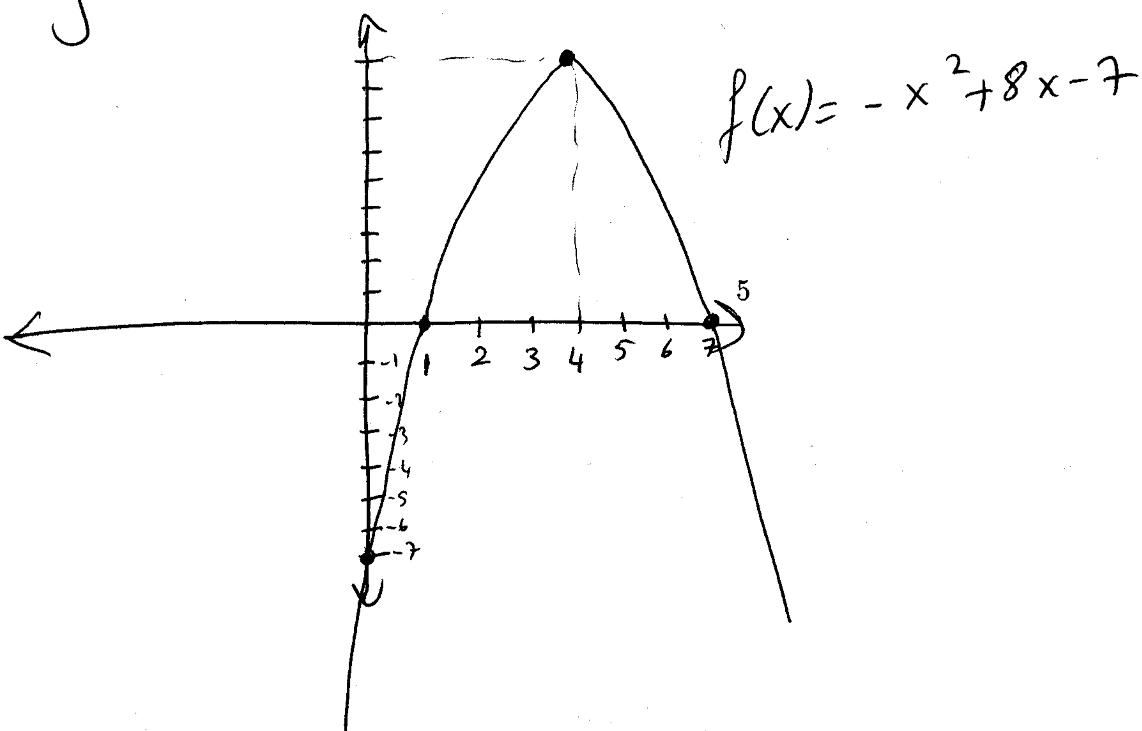
$x=1$ and $x=7$

$$x_{1,2} = \frac{-8 \mp \sqrt{64-4\cdot-1\cdot-7}}{-2}$$

$$x_{1,2} = \frac{-8 \mp \sqrt{36}}{-2}$$

$$x_{1,2} = \frac{-8 \mp 6}{-2} \qquad x_1 = \frac{-14}{-2} = 7 \quad x_2 = \frac{-2}{-2} = 1$$

y-intercept: $f(0) = -7$ $(0,-7)$



5. Find the domain of the following functions

a) (5 points) $f(x) = \dfrac{x^2+7x}{x^2-x-12}$

$x^2 - x - 12 \neq 0$

-4

$+3$

$(x-4)(x+3) \neq 0$

$x \neq 4$ and $x \neq -3$

Domain: $\mathbb{R} - \{-3, 4\}$

or $(-\infty, -3) \cup (-3, 4) \cup (4, +\infty)$

b) (5 points) $g(x) = \sqrt{5x-7}$

$5x - 7 \geq 0$

$x \geq 7/5$

Domain: $[7/5, +\infty)$

c) (5 points) $h(x) = \dfrac{1}{\ln x}$

$\ln x \neq 0$

$e^0 \neq x$

$x \neq 1$

$x > 0$

Domain: $(0, 1) \cup (1, +\infty)$

d) BONUS: (5 points) $k(x) = \sqrt{\log_2 x}$

$\log_2 x \geq 0$

$x \geq 1$

Domain: $[1, +\infty)$

6.

a-)(6 points) Pure water boils at $100C^\circ$ at sea level and at $90C^\circ$ at a hill that is 1000 meters high. Find an equation of the form $T = mx + b$ where T is the boiling temperature of pure water in C° and x is the altitude (yükseklik) in meters.

$(0, 100°c)$

$(1000, 90°c)$

$m_1 = \dfrac{100-90}{0-1000} = \dfrac{+10}{-1000} = -0{,}01$

$T = -0{,}01x + b$

$T = -0{,}01x + 100$

$\boxed{T_1 = -0{,}01x + 100}$

b-)(6 points) Substance A boils at $120C^\circ$ at sea level and at $70C^\circ$ at a hill that is 1000 meters high. Find an equation of the form $T = mx + b$ where T is the boiling temperature of Substance A in C° and x is the altitude (yükseklik) in meters.

$(0, 120°c)$

$(1000, 70°c)$

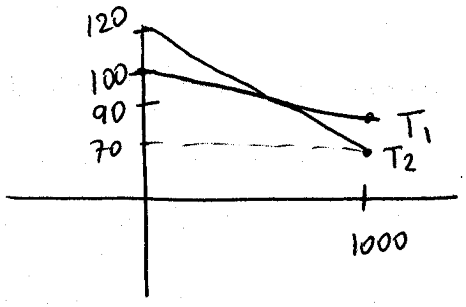
$m_2 = \dfrac{120-70}{0-1000} = \dfrac{5\cancel{0}}{-100\cancel{0}} = -0{,}05$

$T_2 = -0{,}05x + b$

$T_2 = -0{,}05x + 120$

c-)(3 points) Find the altitude in which Substance A and pure water boil at the same temperature.

$T_1 = T_2$



$-0{,}01x + 100 = -0{,}05x + 120$

$+0{,}0\cancel{5}4x = 20$

$x = \dfrac{20}{0{,}04} = \dfrac{2000}{4} = 500$ meters