A list of formulas: $I = Prt$; $A = P(1+rt)$

$$A = P(1+i)^n; \; APY = (1+\tfrac{r}{m})^m - 1$$

$$FV = PMT\tfrac{(1+i)^n-1}{i}, \; PV = PMT\tfrac{[1-(1+i)^{-n}]}{i}, \text{ where } i = \tfrac{r}{m} \text{ and } n = mt$$

1. a-) (5 points) Ali joins a retirement plan of an insurance company when he is 30 years old. If he makes deposits of \$1000 each month for 15 years into this plan which pays 9% compounded monthly, how much money will he have in this account when he is 45 years old?

$$i = \frac{0.09}{12} = 0.0075$$

$$FV = \frac{(1+0.0075)^{180}-1}{0.0075} \times 1000 = \$378{,}405.77$$

b-) (5 points) The retirement age for man in Turkey is 55 years old. If Ali gets fired and stops making payments to this account when he is 45, how much money will he have in the account when he retires at 55? (The money in the account continues to earn interest in this period)

$$A = 378{,}405.77\,(1+0.0075)^{120} =$$

$$\$927{,}607.66$$

c-) (10 points) If Ali decides to withdraw his retirement money by equal month payments in the next 20 years, how much will his monthly withdrawals be?

$$PMT = \frac{PV \times i}{1-(1+i)^{-n}} = \frac{927\,607.66 \times 0.0075}{1-(1.0075)^{-240}}$$

$$= \$8{,}345.93$$

2. Use Gauss Jordan elimination to bring the following augmented matrix into their reduced row echelon form. Write the solution set for the corresponding systems. Find two particular solutions for the system if there exist more than one solution. Determine if the system is consistent, inconsistent, dependent or independent.

(a) (~~20~~ 5 points) $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{array}\right]$ $-R_1 + R_4 \Rightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{array}\right] \quad -R_2 + R_4 \Rightarrow R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{array}\right]$$

no solution

inconsistent system

(b) (~~8~~ 10 points) $\left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & 4 \\ 2 & 3 & 0 & 5 & 3 \\ 4 & 5 & -2 & 9 & 11 \end{array}\right]$ $-2R_1 + R_2 \Rightarrow R_2$

$-4R_1 + R_3 \Rightarrow R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & 4 \\ 0 & 1 & 2 & 1 & -5 \\ 0 & 1 & 2 & 1 & -5 \end{array}\right] \quad -R_2 + R_3 \Rightarrow R_3$$

Consistent

dependent

$$\left[\begin{array}{cccc|c} 1 & 1 & -1 & 2 & 4 \\ 0 & 1 & 2 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad -R_2 + R_1 \Rightarrow R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & 9 \\ 0 & 1 & 2 & 1 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 3x_3 - x_4 + 9$

$x_2 = -2x_3 - x_4 - 5$

$SS = \{(3x_3 - x_4 + 9, -2x_3 - x_4 - 5, x_3, x_4) \mid x_3, x_4 \in \mathbb{R}\}$

Example Solutions : $(9, -5, 0, 0), (8, -8, 1, 1)$

3. (15 points) Solve the following system of linear equation by using the Gauss Jordan elimination.

$$\begin{cases} x_1 + 2x_2 + x_3 &= 8 \\ x_2 + 2x_3 &= 8 \\ 4x_1 + 2x_2 &= 8 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 8 \\ 0 & 1 & 2 & 8 \\ 4 & 2 & 0 & 8 \end{array}\right] \quad -4R_1 + R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 8 \\ 0 & 1 & 2 & 8 \\ 0 & -6 & -4 & -24 \end{array}\right] \quad \begin{array}{l} -2R_2 + R_1 \to R_1 \\ 6R_2 + R_3 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -8 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 8 & 24 \end{array}\right] \quad \frac{1}{8}R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -8 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \begin{array}{l} 3R_3 + R_1 \to R_1 \\ -2R_3 + R_2 \to R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \qquad SS = \{(1, 2, 3)\}$$

4. (10 points) a-) A university offers transportation between city center and campus to its 100 staff members. There are buses and vans available for this service. Each bus can transports 30 people, costs 2000TL to rent and needs two drivers. Each van transports 8 people, costs 150TL and need only one driver. There are 36 drivers employed by the university. Write a mathematical model for minimizing the cost of this transportation project. (DO NOT SOLVE THE SYSTEM)

Let number of buses be b and number of vans v.

Minimize $2000b + 150v$

Subject to

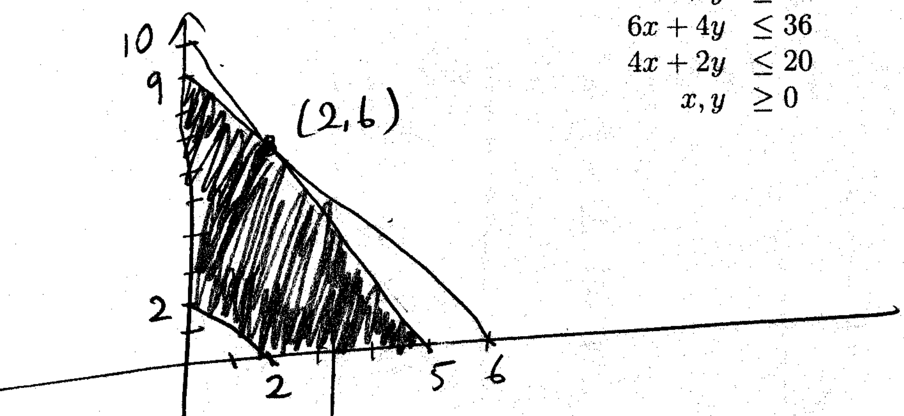
$$30b + 8v \geq 100$$

$$2b + v \leq 36$$

$$b, v \geq 0$$

b-) (20 points) Using the geometric approach, maximize and minimize the objective function $z = 2x + y$ subject to the constraints

$$\begin{aligned} x + y &\geq 2 \\ 6x + 4y &\leq 36 \\ 4x + 2y &\leq 20 \\ x, y &\geq 0 \end{aligned}$$



bounded feasible region

$$\begin{aligned} 6x + 4y &= 36 \\ -2/\ 4x + 2y &= 20 \end{aligned}$$

$$\begin{aligned} 6x + 4y &= 36 \\ -8x - 4y &= -40 \end{aligned}$$

$-2x = -4$

$x = 2$

$6 \times 2 + 4y = 36$

$4y = 24$

$y = 6$

$(2, 6)$

Corner Points	$z = 2x + y$
$(5, 0)$	10
$(2, 0)$	4
$(0, 2)$	2
$(0, 9)$	9
$(2, 6)$	10

Maximum occurs on every point on the line segment between $(5,0)$ and $(2,6)$

Min occurs at $(0,2)$

Max value: 10

Min value: 2^{6}

4. (7 points) a-) Use truth tables to verify the following equivalence

$$p \to (p \wedge q) \Leftrightarrow p \to q$$

p	q	$p \wedge q$	$p \to (p \wedge q)$	$p \to q$
T	T	T	T	T
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

equivalent

b-) (7 points) Determine whether $\neg p \to (p \vee q)$ is tautology, contradiction or contingency.

$\neg p \to (p \vee q) \equiv \neg\neg p \vee (p \vee q) \equiv (p \vee p) \vee q \equiv$

$p \vee q$ Contingency

5. (6 points) A group of 100 people includes 30 who play chess, 90 who play backgammon, and 20 who play both chess and backgammon. How many people in the group play neither game?

$C = \{$ people who play chess $\}$

$B = \{$ people who play backgammon $\}$

$|C \cup B| = |C| + |B| - |C \cap B| = 90 + 30 - 20$

$= 100$

$|(C \cup B)^c| = 100 - 100 = 0$