Fitting a Polynomial to Data

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Given five points $(t_j, y_j),\ j = 1, \ldots, 5$

(1) Fit a quadratic polynomial $q_2(t) := x_3t^2 + x_2t + x_1$.
least squares problem

(2) Fit a cubic polynomial $q_3(t) := x_4t^3 + x_3t^2 + x_2t + x_1$.
least squares problem

(3) Fit a quartic polynomial $q_4(t) := x_5t^4 + x_4t^3 + x_3t^2 + x_2t + x_1$.
polynomial interpolation

Quadratic polynomial $q_2(t) := x_3 t^2 + x_2 t + x_1$

$$\min_{x_1, x_2, x_3 \in \mathbb{R}} \left\| \begin{bmatrix} q_2(t_1) - y_1 \\ q_2(t_2) - y_2 \\ q_2(t_3) - y_3 \\ q_2(t_4) - y_4 \\ q_2(t_5) - y_5 \end{bmatrix} \right\|_2 = \min_{x_1, x_2, x_3 \in \mathbb{R}} \left\| \begin{bmatrix} x_3 t_1^2 + x_2 t_1 + x_1 - y_1 \\ x_3 t_2^2 + x_2 t_2 + x_1 - y_2 \\ x_3 t_3^2 + x_2 t_3 + x_1 - y_3 \\ x_3 t_4^2 + x_2 t_4 + x_1 - y_4 \\ x_3 t_5^2 + x_2 t_5 + x_1 - y_5 \end{bmatrix} \right\|_2$$

Quadratic polynomial $q_2(t) := x_3 t^2 + x_2 t + x_1$

$$\min_{x_1, x_2, x_3 \in \mathbb{R}} \left\| \underbrace{\begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ 1 & t_3 & t_3^2 \\ 1 & t_4 & t_4^2 \\ 1 & t_5 & t_5^2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} - \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}}_{b} \right\|_2$$

Cubic polynomial $q_3(t) := x_4 t^3 + x_3 t^2 + x_2 t + x_1$

$$\min_{x_1,x_2,x_3,x_4 \in \mathbb{R}} \left\| \underbrace{\begin{bmatrix} 1 & t_1 & t_1^2 & t_1^3 \\ 1 & t_2 & t_2^2 & t_2^3 \\ 1 & t_3 & t_3^2 & t_3^3 \\ 1 & t_4 & t_4^2 & t_4^3 \\ 1 & t_5 & t_5^2 & t_5^3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x} - \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}}_{b} \right\|_2$$

Quartic polynomial $q_4(t) := x_5t^4 + x_4t^3 + x_3t^2 + x_2t + x_1$

$$\underbrace{\begin{bmatrix} 1 & t_1 & t_1^2 & t_1^3 & t_1^4 \\ 1 & t_2 & t_2^2 & t_2^3 & t_2^4 \\ 1 & t_3 & t_3^2 & t_3^3 & t_3^4 \\ 1 & t_4 & t_4^2 & t_4^3 & t_4^4 \\ 1 & t_5 & t_5^2 & t_5^3 & t_5^4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}}_{b}$$