Making a Vector Horizontal by Rotators

November 1, 2018

$$v = \begin{bmatrix} x \\ x \\ x \\ \vdots \\ x \\ x \end{bmatrix} \mapsto \begin{bmatrix} x \\ 0 \\ x \\ \vdots \\ x \\ x \end{bmatrix} \mapsto \begin{bmatrix} x \\ 0 \\ 0 \\ \vdots \\ x \\ x \end{bmatrix} \mapsto \quad \dots \quad \mapsto \begin{bmatrix} x \\ 0 \\ 0 \\ \vdots \\ 0 \\ x \end{bmatrix} \mapsto \begin{bmatrix} x \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

kth step

$$v^{(k)} = \begin{bmatrix} a \\ 0 \\ \vdots \\ 0 \\ b \\ x \\ \vdots \\ x \end{bmatrix} \quad \mapsto \quad \begin{bmatrix} \sqrt{a^2+b^2} \\ 0 \\ \vdots \\ 0 \\ 0 \\ x \\ \vdots \\ x \end{bmatrix} = v^{(k+1)} := G_k v^{(k)}$$

$k+1$st entry

$$G_k = \begin{bmatrix} a & & b & \\ & I & & \\ -b & & a & \\ & & & I \end{bmatrix}$$

$k+1$st row

kth step

$$v^{(k)} = \begin{bmatrix} a \\ 0 \\ \vdots \\ 0 \\ b \\ x \\ \vdots \\ x \end{bmatrix} \quad \mapsto \quad \begin{bmatrix} \sqrt{a^2+b^2} \\ 0 \\ \vdots \\ 0 \\ 0 \\ x \\ \vdots \\ x \end{bmatrix} = v^{(k+1)} := G_k v^{(k)}$$

$k+1$st entry

$$G_k = \begin{bmatrix} a/\sqrt{a^2+b^2} & & b/\sqrt{a^2+b^2} & \\ & I & & \\ -b/\sqrt{a^2+b^2} & & a/\sqrt{a^2+b^2} & \\ & & & I \end{bmatrix}$$

$k+1$st row