Computation of All Eigenvalues by the QR Algorithm

% Stage 1

Compute a Hessenberg $H \in \mathbb{C}^{n \times n}$ s.t.

$$H = Q^* A Q$$

for some unitary $Q \in \mathbb{C}^{n \times n}$.

% Stage 2

if H is 1×1 or 2×2 **then**

  $\Lambda \leftarrow$ eigenvalues of H calculated using algebraic formulas

  Return Λ

else

  repeat

    Choose a shift σ

    Compute a QR factorization $H - \sigma I = QR$

    $H \leftarrow RQ + \sigma I$

    if H is of the form $H = \begin{bmatrix} H_1 & H_{12} \\ 0 & H_2 \end{bmatrix}$ for some $H_1 \in \mathbb{C}^{k \times k}$, $H_2 \in \mathbb{C}^{(n-k)\times(n-2)}$ with $k \in [1, n-1]$ **then**

      $\Lambda_1 \leftarrow$ Apply Stage 2 on H_1.

      $\Lambda_2 \leftarrow$ Apply Stage 2 on H_2.

      $\Lambda \leftarrow \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix}$

      Return Λ

    end if

  until

end if