On the Existence of k–homogeneous Latin Bitrades

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(joint work with Behrooz Bagheri Gh.)

Let T be a partial Latin square and L a Latin square such that $T \subseteq L$. Then T is called a Latin trade, if there exists a partial Latin square T^* such that $T^* \cap T = \emptyset$ and $(L \backslash T) \cup T^*$ is a Latin square. We call T^* a disjoint mate of T and the pair (T, T^*) is called a Latin bitrade. A Latin bitrade which is obtained from another one by deleting its empty rows and empty columns, is called a k–homogeneous Latin bitrade, if in each row and each column it contains exactly k elements, and each element appears exactly k times. The number of elements in a Latin trade is referred to as its volume.

Following the earlier work on k–homogeneous Latin bitrades by Cavenagh, Donovan, and Drápal (2003 and 2004) Bean, Bidkhori, Khosravi, and E. S. Mahmoodian (2005) we prove the following,

Theorem. All k–homogeneous Latin bitrades of volume km exist

- for each odd number k and $m \geq k$, and
- for each even number k and $m \geq \min\{(k+p), \frac{3k}{2}\}$, where p is any odd prime number which divides k.

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