Designs and extremal hypergraph problems

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Let $\mathcal{F}$ be a (finite) class of k-uniform hypergraphs, and let $ex(n, \mathcal{F})$ denote its Turan number, i.e., the maximum size of the $\mathcal{F}$-free, n-vertex, k-uniform hypergraphs. In other words, we consider maximal k-hypergraphs satisfying a local constraint. E.g., a Steiner system $S(n, k, t)$ is just a maximum k-hypergraph with no two sets intersecting in t or more elements.

In this lecture old and new Turan type problems are considered, when forbidden class is obtained from coding problems. We emphasize constructions applying algebraic/design theoretic tools with some additional twists. As an example, here is a conjecture from the 1980's.

Let $\mathcal{H}$ be a triple system on n vertices without four distinct members $A, B, C, D \in \mathcal{H}$ such that $A \cap B = C \cap D = \emptyset$ and $A \cup B = C \cup D$, in other words, $\mathcal{H}$ does not have two disjoint pairs with the same union. We conjecture that $|\mathcal{H}| \le \binom{n}{2}$. Equality can be obtained by replacing the 5-element blocks of an $S(n, 5, 2)$ by its 3-subsets.

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