Edge-Extremal Graphs Under Degree and Matching Number Restrictions

Cemil Dibek

Bogazici University

cemil.dibek@boun.edu.tr

(joint work with Tinaz Ekim and Pinar Heggernes)

A graph with an upper bound on its matching number but without a bound on its maximum degree or a graph with an upper bound on its maximum degree but without a bound on its matching number would have infinitely many edges. In order to limit the maximum number of edges of a graph to a finite number, bounds on both maximum degree and matching number are needed. The edge-extremal problem deals with maximizing the number of edges of a graph under restrictions on its maximum degree and matching number. This type of problems are generally studied in the field of Extremal Graph Theory whose main concern is to find extremal graphs that satisfy a certain property. The answer to the edge-extremal problem is known for arbitrary graphs [1]. It is interesting to solve the edge-extremal problem when imposed some structure on the given graphs since the maximum number of edges may change upon narrowing the graph class. The answer when the graphs belong to some chosen graph classes is provided by a recent master thesis [2]. The problem has been answered in that thesis for bipartite graphs, split graphs, disjoint union of split graphs and unit interval graphs. It is observed that star graphs seems to play a central role in the bound on edges. Some open questions have been therefore posed concerning how allowing or disallowing stars affects the bound on the number of edges. In this thesis we provide, to the best of our knowledge, the first results of the edge-extremal problem in claw-free graphs. We find an answer to the change in edge-extremal instances for general graphs when we do not allow claws, which is a special star graph. For this purpose, we develop several claw-free graph constructions and we find the number of edges of an edge-extremal claw-free graph, not only by giving the number itself but also by providing an edge-extremal claw-free graph for each possible case.

References

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[2] Maland, E. (2015). Maximum number of edges in graph classes under degree and matching constraints. Master's thesis. University of Bergen, Norway.

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