Extensions of Equimatchable Graphs

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(joint work with Zakir Deniz and Tatiana Romina Hartinger and Martin Milanič and Mordechai Shalom)

A graph is said to be equimatchable if all its maximal matchings are of the same size. In this work we introduce three extensions of the property of equimatchability and present some initial structural and algorithmic insights about them.
We denote by $\nu(G)$ and $\beta(G)$ the sizes of a maximum matching of a graph G and of a minimum maximal matching of G, respectively.

Consider an edge-weighted graph (G, w) (with $w : E(G) \longrightarrow R^+$) and the greedy algorithm that forms a maximal matching by starting with the empty matching and iteratively adding to it an edge of maximum possible weight. A maximal matching M of G is said to be a greedy matching of (G, w) if it can be chosen by the greedy algorithm. An edge-weighted graph is t-greedy equimatchable if the weight of every greedy matching of it is t, greedy equimatchable if it is t-greedy equimatchable for some t, and strongly greedy equimatchable if it is $\nu(G, w)$-greedy equimatchable, where $\nu(G, w)$ denotes the maximum w-weight of a matching in G. Note that t, if exists, is uniquely determined by (G, w). We give some sufficient conditions for greedy equimatchability.

Given a graph G, we say that a set $S \subseteq V(G)$ is equimatchable (in G) if all maximal matchings of G that cover S are of the same size. We denote by $\eta(G)$ the minimum size of an equimatchable set in G. Clearly, a graph G is equimatchable if and only if $\eta(G) = 0$. We show that computing $\eta(G)$ for a given graph G is APX-hard (in particular, it is NP-hard). However, for each constant k, testing if $\eta(G) \leq k$ is polynomial.

For a graph G, we define $\mu(G) = \nu(G) - \beta(G)$, and say that G is k-quasi-equimatchable if $\mu(G) = k$. Clearly, a graph is equimatchable if and only if it is 0-quasi-equimatchable. We characterize 1-quasi-equimatchable graphs by the existence of a special pair of P_4 and

matching. We also provide necessary and sufficient conditions for a graph to have $\mu(G) \geq k$; this allows us to recognize k-quasi-equimatchable graphs in polynomial time when k is bounded.

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