Existentially Closed Graphs Arising from Combinatorial Designs

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Let n be a positive integer. A graph is called n-existentially closed, or simply n-e.c., if for all disjoint sets of vertices A and B with $|A \cup B| = n$ (one of A or B may be empty), there is a vertex $z \notin A \cup B$ joined to each vertex in A but no vertex in B. Forbes, J. Grannell, and Griggs (Electron. J. Combin., 12(3):#R42, 2005), McKay and Pike (Electron. J. Combin., 14(4):#R70, 2007) have investigated if the block-intersection graph of a BIBD, in particular, an STS, is an n-e.c. graph.

In this talk, we generalize the study of block-intersection graphs to pairwise balanced designs and Steiner quadruple systems. We also characterize the 1-block-intersection graphs of Steiner quadruple systems which are 2-existentially closed.

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