

A First Look Through Fano Kaleidoscopes

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(joint work with Marco Buratti)

Let \mathcal{F} be a set of Fano planes, or 2-(7, 3, 1)-designs, such that the seven lines of each plane are colored with seven different colors c_0, c_1, \dots, c_6 , and that the points of the planes of \mathcal{F} belong to a given v -set \mathcal{P} . We say that \mathcal{F} is a *Fano-Kaleidoscope* of order v (briefly $\text{FK}(v)$) if for any two distinct points x, y of \mathcal{P} and any color $c_i \in \{c_0, c_1, \dots, c_6\}$ there is exactly one Fano plane of \mathcal{F} whose c_i -colored line contains x and y .

This concept can be also studied in the framework of colored designs and edge-colored graph decompositions, see for instance the work by Colbourn and Stinson (1998), Caro, Roditty and Schönheim (1995, 1997, 2002), Adams, Bryant and Jordon (2006), and for whose asymptotic existence we have the important result by Lamken and Wilson (2000).

A necessary condition for the existence of a $\text{FK}(v)$ is that $v \equiv 1 \pmod{6}$. While proving the sufficiency appears to be hard, we will present some first constructions obtained via difference methods.

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