A review of some results on a class of nonlocal wave-type equations

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This talk will be about some results for nonlocal wave type equations. In 1-dimension the equation that will be considered takes the form

$$u_{tt} = (\beta * (u + g(u)))_{xx}$$

where the symbol $*$ denotes the convolution and β is a general kernel function. This equation, related coupled systems and certain 2-d generalizations that we have investigated arise in elasticity models as well in many natural phenomena. Well known equations such as the Improved Boussinesq equation and its generalizations turn out to be specific examples of the nonlocal equation above. Our class also contains integro-differential and difference-differential wave equatins.

We consider the Cauchy (initial value) problem (typically in 1-d)

$$\begin{aligned} u_{tt} &= (\beta * (u + g(u)))_{xx}, \quad x \in \mathbb{R}, \ t > 0, \\ u(x,0) &= \varphi(x), \qquad u_t(x,0) = \psi(x), \quad x \in \mathbb{R} \end{aligned}$$

for initial data in suitable Sobolev spaces. Our results answer basic questions as the local existence, uniqueness and the life span of the solution.