

LECTURE SERIES in GEOMETRIC MEASURE THEORY

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Curvature estimates for surfaces with bounded mean curvature

Colloquium Talk

We consider a surface in \mathbb{R}^3 . The differential of the normal to the surface, known as the second fundamental form A , is a very important tool for studying the geometry of the surface. In this talk I will first describe how estimates on A give us information about the curvature of the surface; in particular when $|A|$ is bounded the surface cannot bend too sharply. Then I will discuss some results concerning estimates for the norm of the second fundamental form, $|A|$, for surfaces with bounded mean curvature (i.e. for which the trace of A is bounded). In particular I will show that for an embedded geodesic disk with bounded L^2 norm of $|A|$, $|A|$ is bounded at interior points, provided that the $W^{1,p}$ norm of its mean curvature is sufficiently small, $p > 2$. This is joint work with Giuseppe Tinaglia.

Introduction to Geometric Measure Theory (GMT)

Introductory Lectures

GMT provides a way of studying objects that are more general (and less smooth) than a manifold.

In this mini course I will introduce the notion of a rectifiable varifold (as a generalization of a submanifold) and define a generalized mean curvature on it which gives rise to the definition of a stationary varifold, which is a generalization of a classical minimal surface. I will discuss some of their properties, the main one being a monotonicity formula about the area ratios. Finally I will describe some well-known theorems, such as compactness and regularity theorems, which are very useful in the study of smooth surfaces as well. Time permitting, I will introduce the notion of a rectifiable current (as a generalization of an orientable submanifold) and mention how these objects help us in the study of area minimization problems.