Stability Analysis of Asset Flow Differential Equations

By Ahmet Duran, May 26, 2011

ABSTRACT

Asset flow differential equations (AFDEs) have been developed and analyzed asymptotically by Caginalp and collaborators since 1989 (see Caginalp and Balenovich, Phil. Trans. R. Soc 1999; Caginalp and Ermentrout, Applied Mathematics Letters 1991; and references contained therein). This important mathematical model may explain various nonlinear behaviors such as overreaction, momentum, bubbles, and crashes in experimental asset markets and real financial markets (see Caginalp, Porter, and Smith, J. Psychol. Finan. Mark. 2000). It incorporates several motivations for buying or selling stock with the finiteness of assets and microeconomic principles.

It is important to understand and classify the behavior of solutions for the dynamical system of nonlinear differential equations. This is more challenging for dimension $n \geq 3$. I studied the stability analysis of the solutions for the dynamical system of nonlinear AFDEs in R^4, in three versions, analytically and numerically (see Duran, Applied Mathematics Letters, 2011). I found the existence of the infinitely many fixed points for the first two versions. I concluded that these versions of AFDEs are structurally unstable systems mathematically by using an extension of the Peixoto Theorem for two-dimensional manifolds to a four dimensional manifold. Moreover, I found that there is no critical point if the chronic discount over the past finite time interval is nonzero for the third version of AFDEs.

It is crucial to analyze the sources of ill-posedness in mathematical modeling. I showed that the existence of multiple roots and that of non-isolated roots are sources of the ill-posedness for the first two versions of AFDEs (see Duran, Applied Mathematics Letters, 2011). I illustrated how to reformulate the problem in order to eliminate any hypersensitivity in the mathematical model. I introduced illustrative examples where analytical and numerical results seem to conflict each other and showed how to reconcile them. The analysis in this work is important for parameter optimization of the related dynamical system of differential equations and exception handling. Moreover, arbitrary perturbations in numerical computation may lead to ill-posedness, especially for such highly nonlinear dynamical systems. Therefore, I suggest using financially meaningful optimal or feasible parameter vectors rather than arbitrary choices. They can be obtained by using nonlinear least-square curve fitting without overfitting (for example, see Duran and Caginalp 2008).