Euler's pentagonal numbers theorem - refinements, variations, and companions

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Euler's celebrated pentagonal numbers theorem is one of the most fundamental in the theory of partitions and q-hypergeometric series. The recurrence formula that it yields is what MacMahon used to compute a table of values of the partition function to verify the deep Hardy-Ramanujan asymptotic formula. On seeing this table, Ramanujan wrote down his spectacular partition congruences. I recently proved two new companions to Euler's theorem in which the role of the pentagonal numbers is replaced by the squares. These companions are deeper in the sense that lacunarity can be achieved even with the introduction of a parameter. One of these companions is deduced from a partial theta identity in Ramanujan's Lost Notebook and the other from a q-hypergeometric identity of George Andrews. We will explain connections between our companions and various classical results such as the Jacobi Triple Product Identity for theta functions and the partition theorems of Sylvester and Fine.

The talk will be accessible to non-experts and graduate students.