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# ON THE NUMBER OF SOLUTIONS OF QUADRATIC EQUATIONS

(joint work with I. Itenberg and V. Kharlamov)

We discuss the classical problem on the number of connected components of an intersection of real quadric in a real projective space. In spite of the fact that all equations are quadratic, we show (by considering a few simple boundary cases) that the problem is much more difficult than it seems and that the 'obvious' bounds that one may conjecture fail.

Our principal results concern an 'advanced' boundary case, namely, intersections of three real quadrics. Let $B_2^0(N)$ be the maximal number of connected components that a regular complete intersection of three real quadrics in $\mathbb{P}^N$ can have. We prove that $B_2^0(N)$ differs at most by one from the maximal number $\operatorname{Hilb}(d)$ of ovals of the submaximal depth $[(N-1)/2]$ of a real plane projective curve of degree $d = N+1$. As a consequence, we obtain the bounds

$$\frac{1}{4}N^2 + O(N) < B_2^0(N) < \frac{3}{8}N^2 + O(N).$$

In particular, our result implies that there is no uniform bound on the number of connected components of an intersection of real quadrics depending only on the codimension of the intersection.