

## Parameters

|              |                                                                                 |
|--------------|---------------------------------------------------------------------------------|
| $\eta$       | boiler efficiency [kJ/kg]                                                       |
| cc           | calorific value of the fuel                                                     |
| a            | consumption parameter for HP generation                                         |
| b            | fixed consumption constant for HP generation                                    |
| $s_{SO_x}$   | $SO_x$ emission parameter [ $Nm^3$ /ton of fuel]                                |
| $s_{GHG}$    | GHG emission parameter [kg/ton of fuel]                                         |
| c            | cost of unit amount of fuel [\$/ton]                                            |
| e            | coefficient of Electricity generation vs. HP steam input                        |
| g            | coefficient of Electricity generation vs. MP-LP steam and condensate generation |
| cpt          | storage capacity of a fuel tank [ton]                                           |
| ssf          | safety stock parameter (defined as a fraction of cpt)                           |
| $p^L, p^U$   | lower and upper limits on fuel purchase [ton/order]                             |
| cpo          | fixed cost of purchasing fuel [\$/order]                                        |
| h            | unit holding cost of fuel [\$/ton of fuel]                                      |
| d            | demand of a certain product [kWh/hr or ton/hr]                                  |
| n            | length of a period [hours]                                                      |
| $s_{SO_x}^U$ | upper limit on $SO_x$ emission [ $Nm^3$ /yr]                                    |
| $s_{GHG}^U$  | upper limit on GHG emission [kg/yr]                                             |
| $\beta$      | cost coefficient the exchange investment                                        |
| $\alpha$     | constant of the fixed cost of exchange investment [\$/]                         |
| ce           | price of unit Electricity [\$/ kWh]                                             |
| $cr_{SO_x}$  | penalty cost of $SO_x$ release [\$/ $Nm^3$ ]                                    |

## **List of Tables**

Table 1. Operating characteristics for the energy system in the 2-company problem.

Table 2. Energy demand in the 2-company problem.

Table 3. Model and solution statistics for the 2-company problem.

Table 4. Summary of the results for the non-integrated solution in the 2-company problem.

Table 5. Summary of the results for the integrated solution in the 2-company problem.

Table 6. Comparison of the results for the 2-company problem.

Table 7. Operating characteristics for the energy system in the 3-company problem.

Table 8. Energy demand in the 3-company problem.

Table 9. Model and solution statistics for the 3-company problem.

Table 10. Summary of the results for the non-integrated solution in the 3-company problem.

Table 11. Summary of the results for the integrated solution in the 3-company problem.

Table 12. Comparison of the results for the 3-company problem.

Table 1. Operating characteristics for the energy system in the 2-company problem.

|                        | Company 1 |           | Company 2 |           |
|------------------------|-----------|-----------|-----------|-----------|
|                        | Fuel 1    | Fuel 2    | Fuel 1    | Fuel 2    |
| $cc_k$                 | 10.50     | 9.65      | 6.65      | 10.20     |
| $I_{ijkt0}$            | 100       | 40        | 120       | 100       |
| $cpt$                  | 120       | 50        | 130       | 110       |
| $s_{SOxk}$             | 7.80      | 1.42      | 1.20      | 5.13      |
| $s_{GHGk}$             | 17        | 5         | 3         | 10        |
| $c_k$                  | 200       | 76        | 83        | 145       |
|                        | Boiler 1  | Boiler 2  | Boiler 1  | Boiler 2  |
| $\eta_{ijk_{fuel1}}$   | 0.590     | 0.575     | 0.560     | 0.565     |
| $\eta_{ijk_{fuel2}}$   | 0.600     | 0.595     | 0.605     | 0.600     |
| $a_{ijk_{MP}}$         | 0.11      | 0.12      | 0.11      | 0.12      |
| $a_{ijk_{EL}}$         | 0.002     | 0.003     | 0.0025    | 0.0028    |
| $X_{ijk_{HP}^u}^{gen}$ | 550       | 550       | 600       | 600       |
| $b_{ijk_{HP}}$         | 0.001     | 0.001     | 0.001     | 0.001     |
| $b_{ijk_{EL}}$         | 0.001     | 0.001     | 0.001     | 0.001     |
|                        | Turbine 1 | Turbine 2 | Turbine 1 | Turbine 2 |
| $e_{ijk_{HP}}$         | 0.150     | 0.175     | 0.160     | 0.170     |
| $e_{ijk_{MP}}$         | 0.070     | 0.080     | 0.070     | 0.075     |
| $e_{ijk_{LP}}$         | 0.009     | 0.010     | 0.012     | 0.010     |
| $X_{ijk_{EL}^u}^{gen}$ | 70        | 60        | 70        | 65        |
| $X_{ijk_{HP}^u}^{in}$  | 900       | 900       | 900       | 900       |
| $X_{ijk_{MP}^u}^{gen}$ | 300       | 300       | 400       | 400       |
| $X_{ijk_{LP}^u}^{gen}$ | 70        | 60        | 70        | 65        |
|                        | HP        | MP        | LP        |           |
| $\alpha_{ij}$          | 0.39      | 0.35      | 0.15      |           |
| $\beta_{ij}$           | 0.11      | 0.10      | 0.04      |           |

Table 2. Energy demand in the 2-company problem.

| Company 1   | Period 1 | Period 2 | Period 3 |
|-------------|----------|----------|----------|
| Electricity | 150      | 200      | 180      |
| HP Steam    | 10       | 13       | 8        |
| MP Steam    | 620      | 423      | 510      |
| LP Steam    | 300      | 260      | 350      |
| Company 2   | Period 1 | Period 2 | Period 3 |
| Electricity | 140      | 180      | 160      |
| HP Steam    | 10       | 15       | 12       |
| MP Steam    | 300      | 345      | 385      |
| LP Steam    | 680      | 500      | 570      |

Table 3. Model and solution statistics for the 2-company problem.

|                                                 | Nonintegrated | Integrated |
|-------------------------------------------------|---------------|------------|
| Number of constraints                           | 627           | 627        |
| Number of variables                             | 665           | 665        |
| Number of nodes in the<br>branch and bound tree | 0             | 0          |
| Number of iterations                            | 100           | 162        |
| CPU time (*sec)                                 | 0.070         | 0.070      |

\* On a PC with Pentium 4 2.6 GHz Processor and 512 MB memory.

Table 4. Summary of the results for the non-integrated solution in the 2-company problem.

| Company  | Unit        | Company  | Unit        | Period | Value  |
|----------|-------------|----------|-------------|--------|--------|
| Company1 | HP          | Company1 | MP          | 1      | 140.57 |
| Company1 | HP          | Company1 | MP          | 3      | 22.11  |
| Company2 | HP          | Company2 | MP          | 1      | 0.54   |
| Company2 | MP          | Company2 | LP          | 1      | 150.34 |
| CompanyU | Electricity | Company1 | Electricity | 1      | 53.20  |
| CompanyU | Electricity | Company1 | Electricity | 2      | 115.11 |
| CompanyU | Electricity | Company1 | Electricity | 3      | 75.94  |
| CompanyU | Electricity | Company2 | Electricity | 1      | 7.98   |
| CompanyU | Electricity | Company2 | Electricity | 2      | 63.03  |
| CompanyU | Electricity | Company2 | Electricity | 3      | 28.10  |

Table 5. Summary of the results for the integrated solution in the 2-company problem.

| Company  | Unit        | Company  | Unit        | Period | Value  |
|----------|-------------|----------|-------------|--------|--------|
| Company1 | LP          | Company2 | LP          | 1      | 89.22  |
| Company1 | LP          | Company2 | LP          | 3      | 89.22  |
| Company2 | HP          | Company2 | MP          | 3      | 9.41   |
| Company2 | MP          | Company1 | MP          | 1      | 151.98 |
| Company2 | MP          | Company1 | MP          | 2      | 151.98 |
| Company2 | MP          | Company1 | MP          | 3      | 151.98 |
| Company2 | MP          | Company2 | LP          | 1      | 165.05 |
| CompanyU | Electricity | Company1 | Electricity | 1      | 42.00  |
| CompanyU | Electricity | Company1 | Electricity | 2      | 128.41 |
| CompanyU | Electricity | Company1 | Electricity | 3      | 74.22  |
| CompanyU | Electricity | Company2 | Electricity | 1      | 8.15   |
| CompanyU | Electricity | Company2 | Electricity | 2      | 48.04  |
| CompanyU | Electricity | Company2 | Electricity | 3      | 28.08  |

Table 6. Comparison of the results for the 2-company problem.

|                         | Non-Integrated | Integrated   | Improvement (%) |
|-------------------------|----------------|--------------|-----------------|
| Total Cost              | 49,019.23      | 47,488.84    | 3.12            |
| SO <sub>x</sub> Release | 2,619,268.07   | 2,346,686.04 | 10.40           |
| GHG Release             | 6,534,323.61   | 5,939,689.22 | 9.10            |

Table 7. Operating characteristics for the energy system in the 3-company problem.

|                            | Company 1 |           | Company 2 |           | Company 2 |           |
|----------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
|                            | Fuel 1    | Fuel 2    | Fuel 1    | Fuel 2    | Fuel 1    | Fuel 2    |
| $cc_k$                     | 10.50     | 9.65      | 6.65      | 10.20     | 11.00     | 12.00     |
| $I_{ijkt0}$                | 100       | 40        | 120       | 100       | 120       | 100       |
| $cpt$                      | 120       | 50        | 130       | 110       | 130       | 110       |
| $s_{SOxk}$                 | 7.80      | 1.42      | 1.20      | 5.13      | 4.83      | 2.62      |
| $s_{GHGk}$                 | 17        | 5         | 3         | 10        | 11        | 8         |
| $c_k$                      | 200       | 76        | 83        | 145       | 94        | 102       |
|                            | Boiler 1  | Boiler 2  | Boiler 1  | Boiler 2  | Boiler 1  | Boiler 2  |
| $\eta_{ijk_{fuel1}}$       | 0.590     | 0.575     | 0.560     | 0.565     | 0.580     | 0.595     |
| $\eta_{ijk_{fuel2}}$       | 0.600     | 0.595     | 0.605     | 0.600     | 0.570     | 0.605     |
| $a_{ijk_{MP}}$             | 0.11      | 0.12      | 0.11      | 0.12      | 0.1150    | 0.1210    |
| $a_{ijk_{EL}}$             | 0.002     | 0.003     | 0.0025    | 0.0028    | 0.0026    | 0.0029    |
| $X_{ijk_{HP}^u}^{l_{gen}}$ | 550       | 550       | 600       | 600       | 600       | 600       |
| $b_{ijk_{HP}}$             | 0.001     | 0.001     | 0.001     | 0.001     | 0.001     | 0.001     |
| $b_{ijk_{EL}}$             | 0.001     | 0.001     | 0.001     | 0.001     | 0.001     | 0.001     |
|                            | Turbine 1 | Turbine 2 | Turbine 1 | Turbine 2 | Turbine 1 | Turbine 2 |
| $e_{ijk_{HP}}$             | 0.150     | 0.175     | 0.160     | 0.170     | 0.160     | 0.170     |
| $e_{ijk_{MP}}$             | 0.070     | 0.080     | 0.070     | 0.075     | 0.070     | 0.075     |
| $e_{ijk_{LP}}$             | 0.009     | 0.010     | 0.012     | 0.010     | 0.012     | 0.010     |
| $X_{ijk_{EL}^u}^{l_{gen}}$ | 70        | 60        | 70        | 65        | 70        | 65        |
| $X_{ijk_{HP}^u}^{l_{in}}$  | 900       | 900       | 900       | 900       | 900       | 900       |
| $X_{ijk_{MP}^u}^{l_{gen}}$ | 300       | 300       | 400       | 400       | 400       | 400       |
| $X_{ijk_{LP}^u}^{l_{gen}}$ | 70        | 60        | 70        | 65        | 70        | 65        |
|                            | HP        | MP        | LP        |           |           |           |
| $\alpha_{jj'}$             | 0.39      | 0.35      | 0.15      |           |           |           |
| $\beta_{jj'}$              | 0.11      | 0.10      | 0.04      |           |           |           |

Table 8. Energy demand in the 3-company problem.

| Company 1   | Period 1 | Period 2 | Period 3 |
|-------------|----------|----------|----------|
| Electricity | 150      | 200      | 180      |
| HP Steam    | 10       | 13       | 8        |
| MP Steam    | 620      | 423      | 510      |
| LP Steam    | 300      | 260      | 350      |
| Company 2   | Period 1 | Period 2 | Period 3 |
| Electricity | 140      | 180      | 160      |
| HP Steam    | 10       | 15       | 12       |
| MP Steam    | 300      | 345      | 385      |
| LP Steam    | 680      | 500      | 570      |
| Company 3   | Period 1 | Period 2 | Period 3 |
| Electricity | 150      | 170      | 170      |
| HP Steam    | 11       | 14       | 13       |
| MP Steam    | 320      | 350      | 440      |
| LP Steam    | 300      | 340      | 450      |

Table 9. Model and solution statistics for the 3-company problem.

|                                           | Non-Integrated | Integrated |
|-------------------------------------------|----------------|------------|
| Number of constraints                     | 993            | 993        |
| Number of variables                       | 1,042          | 1,042      |
| Number of nodes in the branch<br>and tree | 0              | 0          |
| Number of iterations                      | 166            | 371        |
| CPU time (*sec)                           | 0.080          | 0.080      |

\* On a PC with Pentium 4 2.6 GHz Processor and 512 MB memory.



Table 10. Summary of the results for the non-integrated solution in the 3-company problem.

| Company  | Unit        | Company  | Unit        | Period | Value  |
|----------|-------------|----------|-------------|--------|--------|
| Company1 | HP          | Company1 | MP          | 1      | 140.57 |
| Company1 | HP          | Company1 | MP          | 3      | 22.11  |
| Company2 | MP          | Company2 | LP          | 1      | 150.23 |
| CompanyU | Electricity | Company1 | Electricity | 1      | 53.20  |
| CompanyU | Electricity | Company1 | Electricity | 2      | 115.11 |
| CompanyU | Electricity | Company1 | Electricity | 3      | 75.94  |
| CompanyU | Electricity | Company2 | Electricity | 1      | 7.96   |
| CompanyU | Electricity | Company2 | Electricity | 2      | 63.03  |
| CompanyU | Electricity | Company2 | Electricity | 3      | 28.1   |
| CompanyU | Electricity | Company3 | Electricity | 1      | 66.73  |
| CompanyU | Electricity | Company3 | Electricity | 2      | 77.44  |
| CompanyU | Electricity | Company3 | Electricity | 3      | 51.25  |

Table 11. Summary of the results for the integrated solution in the 3-company problem.

| Company  | Unit        | Company  | Unit        | Period | Value  |
|----------|-------------|----------|-------------|--------|--------|
| Company2 | HP          | Company1 | HP          | 1      | 126.82 |
| Company2 | HP          | Company1 | HP          | 2      | 126.82 |
| Company2 | HP          | Company1 | HP          | 3      | 101.38 |
| Company2 | MP          | Company2 | LP          | 1      | 25.18  |
| Company2 | LP          | Company1 | LP          | 2      | 75.18  |
| Company3 | HP          | Company3 | MP          | 1      | 25.58  |
| Company3 | MP          | Company1 | MP          | 1      | 278.00 |
| Company3 | MP          | Company1 | MP          | 2      | 278.00 |
| Company3 | MP          | Company1 | MP          | 3      | 149.02 |
| Company3 | MP          | Company3 | LP          | 3      | 41.44  |
| Company3 | LP          | Company1 | LP          | 1      | 94.57  |
| Company3 | LP          | Company1 | LP          | 2      | 76.40  |
| Company3 | LP          | Company2 | LP          | 1      | 54.82  |
| Company3 | LP          | Company2 | LP          | 3      | 6.38   |
| CompanyU | Electricity | Company1 | Electricity | 1      | 81.28  |
| CompanyU | Electricity | Company1 | Electricity | 2      | 166.99 |
| CompanyU | Electricity | Company1 | Electricity | 3      | 87.97  |
| CompanyU | Electricity | Company2 | Electricity | 1      | 8.18   |
| CompanyU | Electricity | Company2 | Electricity | 2      | 50.07  |
| CompanyU | Electricity | Company2 | Electricity | 3      | 28.18  |
| CompanyU | Electricity | Company3 | Electricity | 1      | 18.30  |
| CompanyU | Electricity | Company3 | Electricity | 2      | 38.30  |
| CompanyU | Electricity | Company3 | Electricity | 3      | 38.30  |

Table 12. Comparison of the results for the 3-company problem.

|                         | Non-Integrated | Integrated   | Improvement (%) |
|-------------------------|----------------|--------------|-----------------|
| Total Cost              | 73,773.87      | 71,953.60    | 2.47            |
| SO <sub>x</sub> Release | 4,100,000.00   | 3,527,231.44 | 13.97           |
| GHG Release             | 10,435,585.95  | 9,434,073.34 | 9.60            |