KOÇ UNIVERSITY
College of Arts and Sciences
Department of Physics

Course: MATH503 Applied Mathematics
Credits: 3
Semester: Fall 2003
Instructor: Professor **Tekin Dereli**

1. Midterm Exam: 10 November 2003, 14.00-15.15

Question: 1 (15 points) Given the vectors $\vec{r}_1 = \vec{i} + a\vec{j} + \vec{k}$, $\vec{r}_2 = b\vec{i} + a\vec{j} + b\vec{k}$, $\vec{r}_3 = b\vec{i} - b\vec{k}$, determine all the values of a and b for which they are linearly dependent.

Question: 2 (20 points) Calculate the circumference of the cardioid $r = a(1 + cos\theta)$. Sketch the graph of this curve.

Question: 3 (20 points) State Green's theorem and use it to find the area under one arch of the cycloid

$$\vec{r}(t) = a(t - sin\theta)\vec{i} + a(1 - cos\theta)\vec{j} \ , \ 0 \leq t \leq 2\pi \, .$$

Question: 4 (20 points) Derive the equation of a circle that passes through the points $P_1 : (1,2)$, $P_2 : (2,3)$, $P_3; (3,1)$. Use matrix methods.

Question: 5 (25 points) A rigid 3-body system is given by $m_1 = 1$ at the point $(1,1,-2)$, $m_2 = 2$ at the point $(-1,-1,0)$, $m_3 = 1$ at the point $(1,1,2)$. The components of the moment of inertia tensor are

$$I_{xx} = \sum_{j=1}^{3} m_j ({y_j}^2 + {z_j}^2), \dots ,$$

$$I_{xy} = I_{yx} = -\sum_{j=1}^{3} m_j x_j y_j, \dots .$$

i. Calculate the moment of inertia matrix

$$I = \begin{pmatrix} 12 & -4 & 0 \\ -4 & 12 & 0 \\ 0 & 0 & 8 \end{pmatrix} .$$

ii. Diagonalize the moment of inertia matrix, obtaining the eigenvalues and the principal axes of rotation (as normalized eigenvectors).