

KOÇ UNIVERSITY
College of Arts and Sciences
Department of Physics

Course: MATH503 Applied Mathematics

Credits: 3

Semester: Fall 2003

Instructor: Professor **Tekin Dereli**

2. Midterm Exam: 8 December 2003, 17.00-18.15

Question: 1 (25 points) Find the Fourier series expansion of the function

$$f(x) = \begin{cases} 1 & , \quad -\pi/2 < x < \pi/2 \\ 0 & \textit{otherwise} \end{cases}$$

that is assumed to have period 2π .

Question: 2 (25 points) Determine the Fourier transform of the function $f(x) = e^{-x^2/2}$.

Reminder:

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-ikx} dx.$$

Question: 3 (25 points) Solve the following p.d.e. by separation of variables:

$$x \frac{\partial^2 u}{\partial x \partial y} + 2yu = 0.$$

Question: 4 (25 points) Find the solution of the 1-dimensional heat equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

in integral form

$$u(x, t) = \int_0^{\infty} (A(p) \cos px + B(p) \sin px) e^{-c^2 p^2 t} dp$$

subject to the initial condition $u(x, 0) = f(x)$ where

$$f(x) = \begin{cases} 1 & , \quad |x| < 1 \\ 0 & \textit{otherwise.} \end{cases}$$