Comp 106, HW1, Fall 2019

The problems below are mostly from the first chapter of the 6th edition of your textbook. The same problems (or very similar ones) are also available in the 7th and 8th editions.

Only the first 4 problems (Question 1, Question 2, Question 3 and Question 4) will be graded. Do not submit the solutions for the remaining problems (though you are expected to solve them).

Please provide formal justification for all your answers to the following 4 questions in order to get full credit.

Question 1

Determine whether $\forall x (P(x) \leftrightarrow Q(x))$ and $\forall x \, P(x) \leftrightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

Question 2

Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "x is a duck," "x is one of my poultry," "x is an officer," and "x is willing to waltz," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.

a) No ducks are willing to waltz.
b) No officers ever decline to waltz.
c) All my poultry are ducks.
d) My poultry are not officers.
e) Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

   Explain your answer.

Question 3

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in

simple English. (Do not simply use the words "It is not the case that.")

a) No one has lost more than one thousand dollars playing the lottery.
b) There is a student in this class who has chatted with exactly one other student.
c) No student in this class has sent e-mail to exactly two other students in this class.
d) Some student has solved every exercise in this book.
e) No student has solved at least one exercise in every section of this book.

In Question 3, use the following predicates (or propositional functions):

(a) *Q(x,y)*: "Person *x* has lost *y* dollars playing the lottery"
The universe of discourse for *x* is the set of all people.
The universe of discourse for *y* is the set of all numbers.

(b) *Q(x,y)*: "Student *x* has chatted with student *y*"
The universe of discourse for *x* and *y* is the set of students in this class.

(c) *Q(x,y)*: "Student *x* has sent email to student *y*"
The universe of discourse for *x* and *y* is the set of students in this class.

(d) *Q(x,y)*: "Student *x* solved exercise *y*"
   P(y): "Exercise *y* is in the book"
The universe of discourse for *x* is the set of all students.
The universe of discourse for *y* is the set of all exercises.

(e) *R(x,y,z)*: "Student *x* has solved exercise *y* in section *z* of the book"

The universe of discourse for *x* is the set of all students.
The universe of discourse for *y* is the set of exercises in the book.
The universe of discourse for *z* is the set of sections in the book.

Question 4

Suppose the following two propositions are true (hence they are given):
i) "If you work hard, you can pass this course", and
ii) "You do not work hard".
Based on these given two statements, the following is then concluded: "You cannot pass this course".

  a) Write down the reasoning that leads to this conclusion in terms of logical operators as if it is a **rule of inference**.
  b) Show formally whether or not this is a correct way of reasoning.

Hint: Let p: "You work hard" and q: "You can pass this course".

You are not supposed to submit any solutions for the remaining problems (though you are expected solve them)

Question 5

Is the assertion "This statement is false" a proposition?

Question 6

Are these system specifications consistent? "If the file system is not locked, then new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then new messages will be sent to the message buffer. New messages will not be sent to the message buffer."

Question 7

Show that $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$ are logically equivalent.

Question 8

Express the negation of these propositions using quanti-
fiers, and then express the negation in English.
a) Some drivers do not obey the speed limit.
b) All Swedish movies are serious.
c) No one can keep a secret.
d) There is someone in this class who does not have a
   good attitude.

In Question 3, use the following predicates (or propositional functions):

(a) *P(x)*: "*x* obeys the speed limit"
   Q(x): "*x* is a driver"
The universe of discourse is the set of all people.

(b) *P(x)*: "*x* is serious"
   Q(x): "*x* is Swedish"
The universe of discourse is the set of all movies.

(c) *P(x)*: "*x* can keep a secret"
The universe of discourse is the set of all people.

(d) *P(x)*: "*x* is in this class"
   Q(x): "*x* has a good attitude"
The universe of discourse is the set of all students.

Question 9

Let $A(x)$ and $B(x, y)$ be the predicates:
   $A(x)$: "x passes the exam,"
   $B(x, y)$: "x answers question y correctly,"
where the **universe of discourse** for x is the set of all students in this class,
and for y, the set of all questions in the exam. Express the following statement
using quantifiers, logical connectives and the given predicates:
"No one in this class can pass the exam if (s)he does not answer at least one
question in the exam correctly."
Then **negate** the expression you have found, so that no negation is to the left
of a quantifier. Finally express the negation in simple English.