**Comp106, HW2, Fall 2019**
(The problems below are all from the 3rd chapter of the 6th edition of your textbook. The same problems (or very similar ones) are also available in the 7th and 8th editions).

Only the first 4 problems (Question 1, Question 2, Question 3 and Question 4) will be graded. Do not submit the solutions for the remaining problems (though you are expected to solve them).

Please provide formal justification for all your answers to the following questions in order to get full credit.

**1.** Use the definition of big-$O$ notation "$f(x)$ is $O(g(x))$" to show that $2^x + 65$ is $O(4^x)$.

**2.** Suppose that $f(x)$ is $O(g(x))$. Does it follow that $2^{f(x)}$ is $O(2^{g(x)})$? Use the definition of big-$O$ notation.

**3.** Show that if $5x^2$ is $\Omega(f(x))$ then $x^3$ is also $\Omega(f(x))$. Use the definition of big-$O$ or big-$\Omega$ notation.

**4.** The **binary insertion sort** algorithm (given below as pseudocode) sorts an input list of $n$ numbers by making use of the binary search algorithm. Write down the complexity function of this sorting algorithm by counting only the comparison operations used. Take also into account the comparisons used to implement the for loops involved in the algorithm. Then use this function to give a big-$O$ estimate of the complexity of the binary insertion sort. (Note that counting whether only comparison operations or others as well does not change the big-$O$ estimate.) Give also a big-$\Theta$ estimate. (Hint: Use the identity $\log ab = \log a + \log b$.)

```
procedure binary insertion sort(a_1, a_2, ..., a_n:
    real numbers with n ≥ 2)
for j := 2 to n
    {binary search for insertion location i}
    left := 1
    right := j − 1
    while left < right
        middle := ⌊(left + right)/2⌋
        if a_j > a_middle then left := middle + 1
        else right := middle
    if a_j < a_left then i := left else i := left + 1
    {insert a_j in location i by moving a_i through a_{j−1}
        toward back of list}
    m := a_j
    for k := 0 to j − i − 1
        a_{j−k} := a_{j−k−1}
    a_i := m
{a_1, a_2, ..., a_n are sorted}
```

**You are not supposed to submit any solutions for the remaining problems (though you are expected solve them)**

**5.** Let $f(x) = 5 \log x$. Is $f(x)$ $\Omega(x)$? Is $f(x)$ $\Theta(x)$? Explain your answers.

**6.** Show that $x^3$ is $O(x^4)$ but that $x^4$ is not $O(x^3)$.

**7.** Give a big-$O$ estimate for $f(n) = (n^n + n2^n + 5^n)(n! + 5^n + n^n)$. For the function $g$ in your estimate "$f(x)$ is $O(g(x))$", use a simple function $g$ of the smallest possible order.

**8.** Show that $2x^2 + x - 7$ is $\Theta(x^2)$.

**9.** Show that $\log_{10} x$ is $\Theta(\log_2 x)$.

**10.** Suppose that $f(x)$, $g(x)$, and $h(x)$ are functions such that $f(x)$ is $\Theta(g(x))$ and $g(x)$ is $\Theta(h(x))$. Show that $f(x)$ is $\Theta(h(x))$.