Comp-106, Fall 2019, Questions for HW#5

Only Question 1, Question 2a, Question 3 and Question 4 will be graded. Do not submit the solutions for the remaining problems (though you are expected to solve them).

(Most of the problems below are very similar to or the same as the questions from the relevant chapters of your textbook.)

Please provide formal justification for all your answers to the following questions in order to get full credit.

1. Let R be a symmetric relation. Show that R^n is also symmetric for all positive integers n.

2. Use Algorithm 1 of Section 8.4 (the one also given in lecture notes) to find the transitive closures, hence the connectivity relations, of the following relations on $\{a, b, c, d, e\}$:

a) $\{(b,c), (b,e), (c,e), (d,a), (e,b), (e,c)\}$ **(Submit only this part.)**

b) $\{(a,e), (b,a), (b,d), (c,d), (d,a), (d,c), (e,a), (e,b), (e,c), (e,e)\}$ **(Don't submit this part.)**

3. Do we necessarily get an equivalence relation when we form the symmetric closure of the reflexive closure of the transitive closure of a relation? Explain your answer.

4. Find a compatible total order for the divisibility relation on the set $\{1, 2, 3, 6, 8, 12, 24, 36\}$. You need to draw the Hasse diagram first.

You're not supposed to submit any solutions for the remaining problems (though you're expected solve them)

5. a) Let R be the relation on the set of functions from Z^+ to Z^+ such that (f, g) belongs to R iff f is $\Theta(g)$. Show that R is an equivalence relation.

b) Describe the equivalence class containing $f(n) = n^2$ for the equivalence class of part (a).

6. Show that $R = \{(x, y) \mid x–y \in Q\}$ is an equivalence relation on the set of real numbers, where Q denotes the set of rational numbers. What are $[1]_R$, $[1/2]_R$ and $[\pi]_R$?

The following two questions are from previous years' final exams:

7. Let R be the relation on the set of functions from Z^+ to Z^+ such that

$$(f, g) \in R \longleftrightarrow (f = g) \vee (f \text{ is } O(g) \wedge f \text{ is not } \Theta(g))$$

a) Show that R is a partial order relation.

b) Draw the Hasse diagram and find a compatible total order for the poset (S, R), where $S = \{n, 3n, n^2, n^2 + n, \log n, n^2 + \log n, n \log n, n \log n + 4, 2^n, n!\}$.

8. Consider the relation R on the set $A = \{a, b, c, d, e\}$ with the following matrix representation:

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find the matrix representation for the smallest equivalence relation R_e on set A containing R.

(b) Find the equivalence class of b and the equivalence class of e with respect to R_e.

(c) Draw the graph representation of the relation R_e.