Comp106
Fall Term, 2019

Homework 6

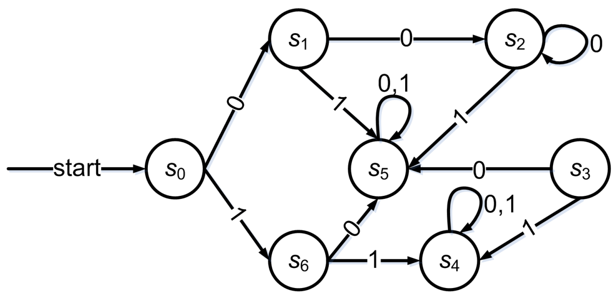
Most of the problems below are very similar to or the same as the questions in the Modeling Computation chapter of your textbook.

1 Construct phrase-structure grammars to generate each of these sets.
a) $\{01^{2n} | n \geq 0\}$
b) $\{0^n 1^{2n} | n \geq 0\}$
c) $\{0^n 1^m 0^n | n \geq 0 \text{ and } m \geq 0\}$

2 Construct a finite-state machine for entering a security code into an automatic teller machine (ATM) that implements these rules: A user enters a string of four digits. If the user enters the correct four digits of the password, the ATM displays a welcome string. When the user enters an incorrect string of four digits, the ATM displays a screen that informs the user that an incorrect password was entered. If a user enters the incorrect password three times, the account is locked. Here, assume that the correct password is 5538.

3 Construct a finite-state machine that models a newspaper vending machine that has a door that can be opened only after either three dimes (and any number of other coins) or a quarter and a nickel (and any number of other coins) have been inserted. Once the door can be opened, the customer opens it and takes a paper, closing the door. No change is ever returned no matter how much extra money has been inserted. The next customer starts with no credit.

4 A state s' in a finite-state machine is said to be **reachable** from state s if there is an input string x such that $f(s, x) = s'$. A state s is called **transient** if there is no nonempty input string x with $f(s, x) = s$. A state is called a **sink** if $f(s, x) = s$ for all input strings x. Answer these questions about the finite-state machine with the state diagram illustrated here.



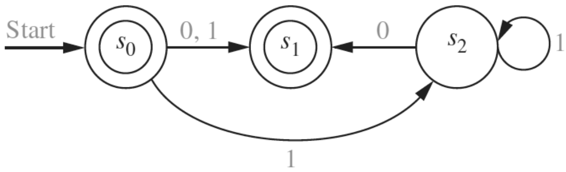
a) Which states are reachable from s_0?
b) Which states are reachable from s_2?
c) Which states are transients?
d) Which states are sinks?

5 Construct a deterministic finite-state automaton that recognizes the set of bit strings that start with 1 and end with 1.

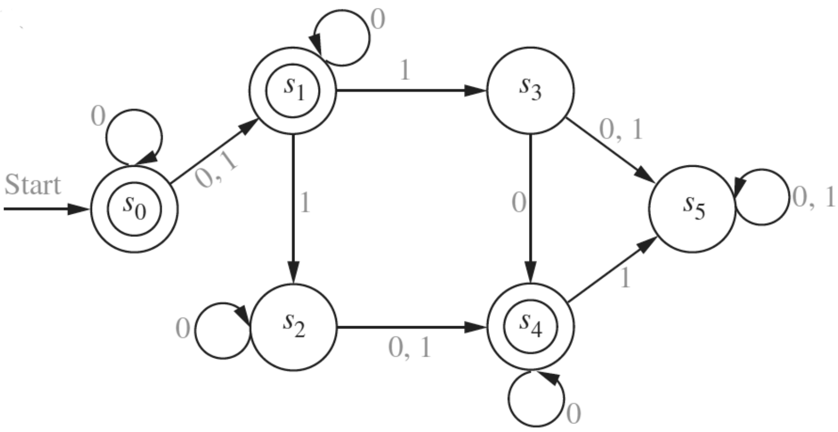
6 Construct a deterministic finite-state automaton with four states that recognizes the set of bit strings containing an even number of 1s and an odd number of 0s.

7 Construct a deterministic finite-state automaton that recognizes the language generated by the grammar $G = (V, T, S, P)$ with vocabulary $V = \{a, b, A, B, S\}$, terminals $T = \{a, b\}$, start symbol S, and production rules $P = \{S \to \lambda, S \to aA, A \to bB, B \to ab\}$.

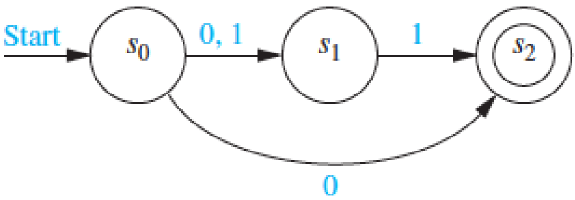
8 Find the language recognized by the given nondeterministic finite-state automaton.



9 Find the language recognized by the given nondeterministic finite-state automaton.



10 Find a deterministic finite-state automaton that recognizes the same language as the non-deterministic finite-state automaton below.



11 Construct a Turing machine with tape symbols a, b and B that, given a string as input, replaces all a symbols on the tape with b and does not change any of the b symbols on the tape. Note that B represent blank.

12 Run the following Turing Machine on input "BB111111BB" and show the state at each step, along with the tape position and tape contents. Which function does this machine compute? Note that the starting state is s_0. Also note that in unary representation, a number n is represented using $(n+1)$ 1s. Hence 0 is represented by '1', 1 is represented by '11', 2 is represented by '111', and so on. (Hint: Therefore, your input is the unary representation of the number 5).

$(s_0, 1 \to s_1, 1, R), (s_1, 1 \to s_2, 1, R), (s_2, 1 \to s_3, 1, R), (s_3, 1 \to s_4, 1, R), (s_4, 1 \to s_4, 1, R), (s_4, B \to s_5, B, L), (s_5, 1 \to s_6, B, L), (s_6, 1 \to s_7, B, L), (s_7, 1 \to s_7, B, R)$.

13 Construct a Turing machine that recognizes the set of all bit strings that end with a 0.