Constrained and Distributed Optimal Control

Francesco Borrelli

University of California

Berkeley, USA

Honeywell Laboratories

Ford Motor Company

Challenges

Hybrid Control Design

&

Distributed Control for Large Scale Systems

Challenges

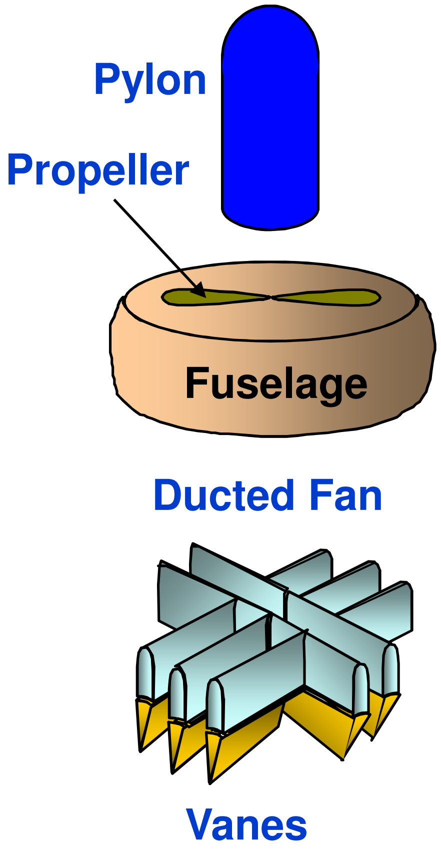
Hybrid Control Design

Switched Linear Systems

Constraint Satisfaction

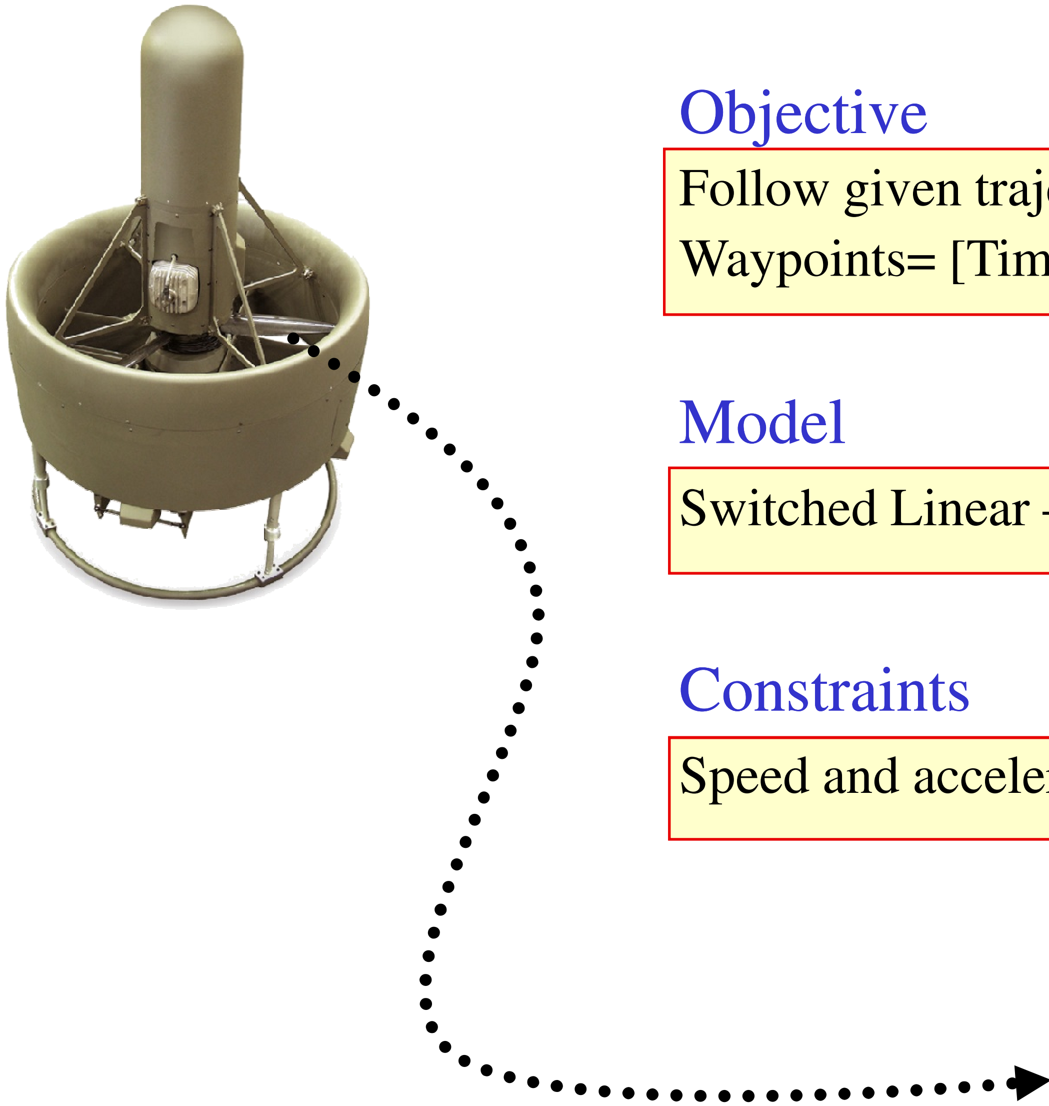
Organic Air Vehicle

Dale Swanson et al.



At high level: Constrained Switched Linear System
External Switch Selects Mode of Operation

OAV Autonomous Flight



Objective

Follow given trajectories.
Waypoints= [Time,Space]

Model

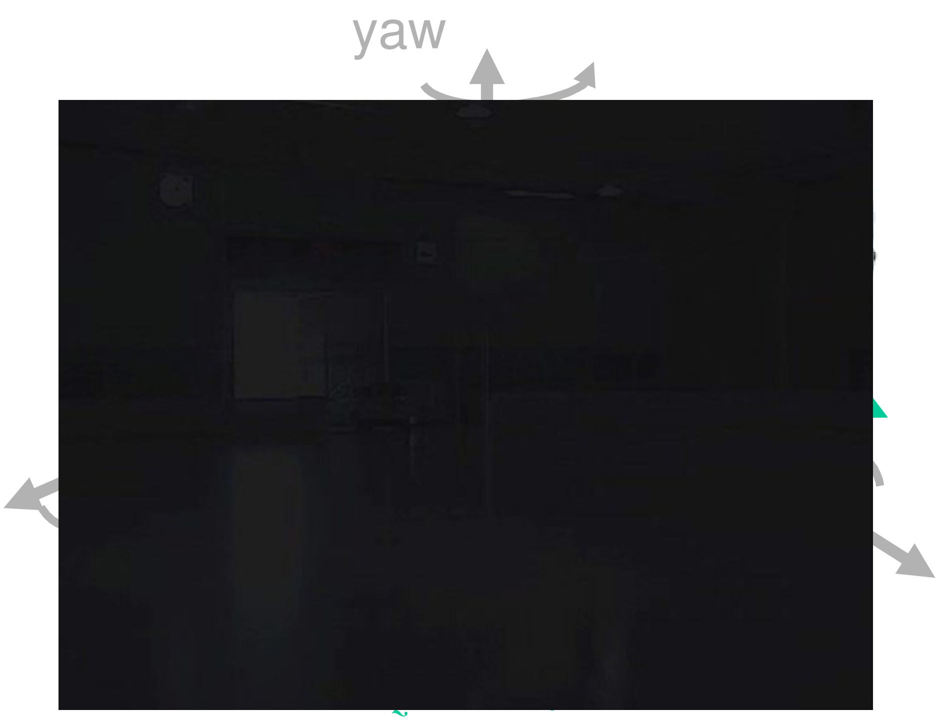
Switched Linear – External Switch

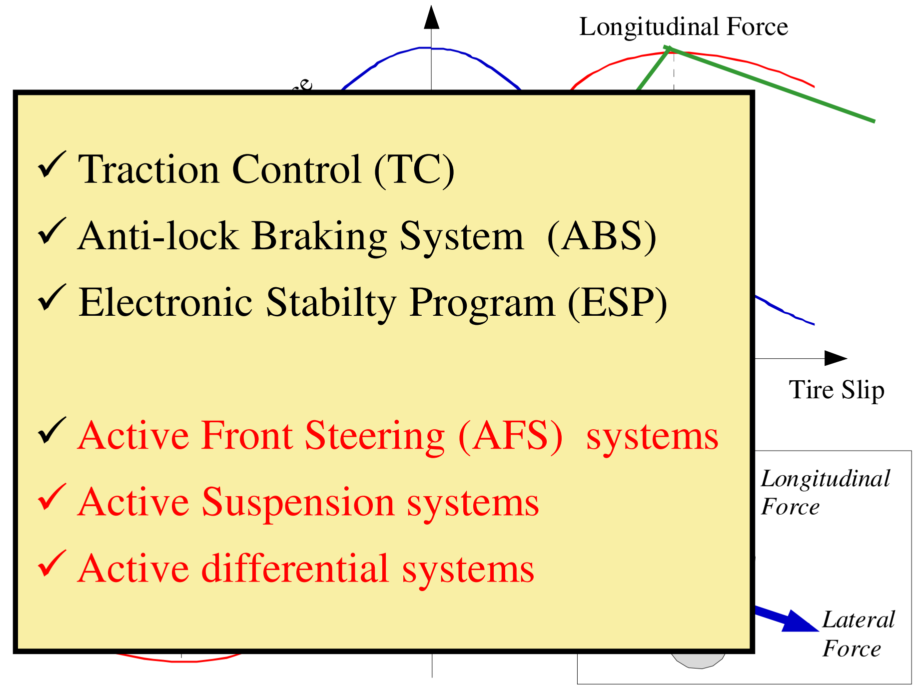
Constraints

Speed and acceleration function of mode

Vehicle Dynamics Control

A driver aid for atypical road conditions, such as slippery, windy an bumpy roads





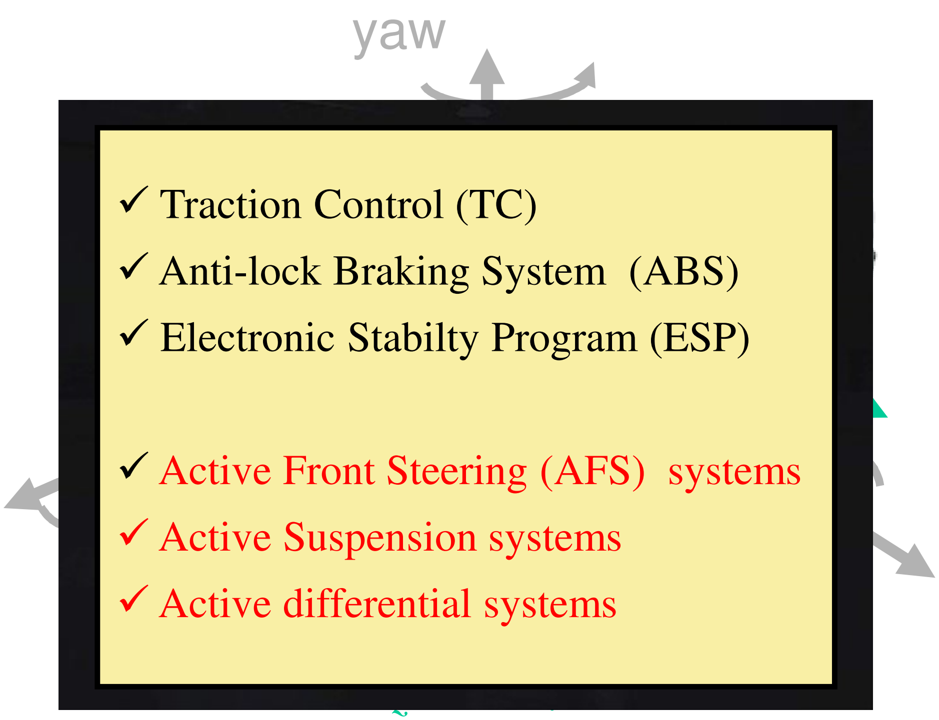
- ✓ Traction Control (TC)
- ✓ Anti-lock Braking System (ABS)
- ✓ Electronic Stabilty Program (ESP)

- ✓ Active Front Steering (AFS) systems
- ✓ Active Suspension systems
- ✓ Active differential systems

Nonlinear (Piece-wise linear) and Constrained System

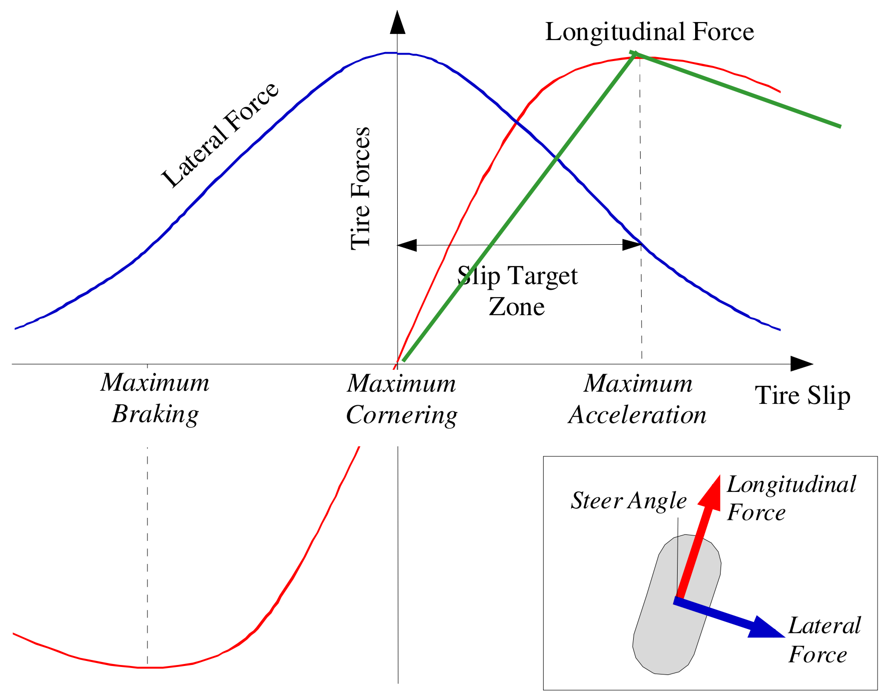
Vehicle Dynamics Control

A driver aid for atypical road conditions, such as slippery, windy an bumpy roads



- ✓ Traction Control (TC)
- ✓ Anti-lock Braking System (ABS)
- ✓ Electronic Stabilty Program (ESP)

- ✓ Active Front Steering (AFS) systems
- ✓ Active Suspension systems
- ✓ Active differential systems



Nonlinear (Piece-wise linear) and Constrained System

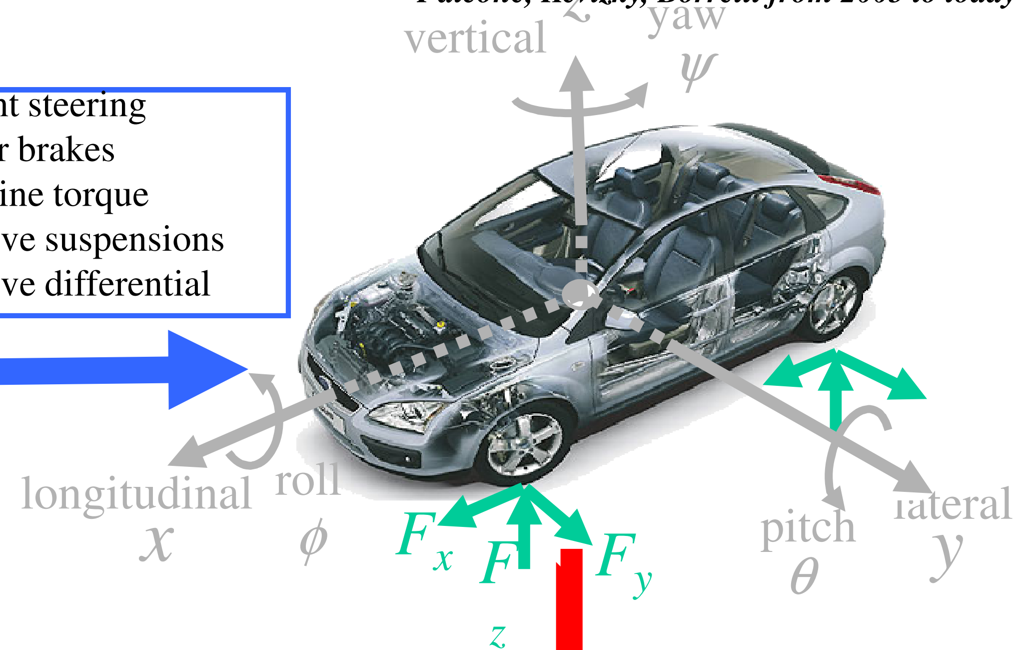
Ford Motor Company

Integrated VDC via MPC

Falcone, Kevizky, Borrelli from 2003 to today

MIMO controller integrating local and global measurements coming from GPS, cameras, infrared and radar

- ✓ Front steering
- ✓ Four brakes
- ✓ Engine torque
- ✓ Active suspensions
- ✓ Active differential



- ✓ . Longitudinal, lateral and vertical velocity/accelerations
- ✓ Yaw, roll and pitch angles/rates
- ✓ Position and velocity in a global frame

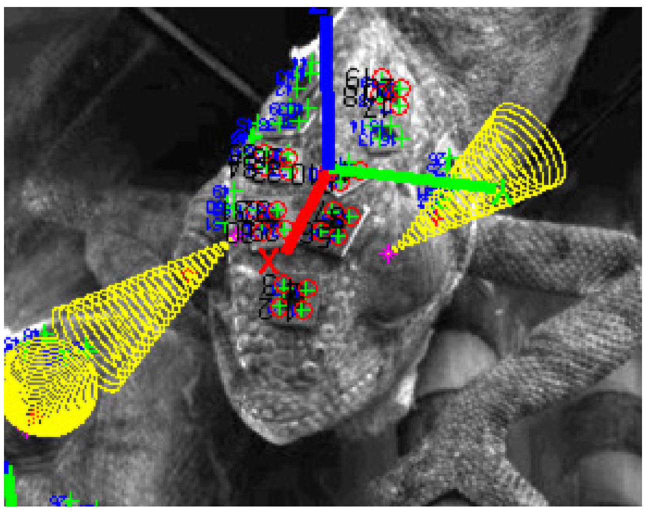
Controlling Yaw, Roll, Pitch, Vertical, Lateral and Longitudinal Dynamics via Multiple Input

Enabling path following capabilities

Davor Hrovat, Jahan Asgari, Eric Tseng, Mike Fodor

Ford Motor Company

Chameleon Visual Tracking



Objective

Tracking of a moving prey

Model

PTZ camera: Linear

Prey: Linear point mass

Constraints

Pan Tilt and Zoom constraints

Prey in tracking window ∀ unknown bounded accelerations

Common Problem Features

- Objective
  - Minimization of performance index
- Models
  - Linear, Uncertain
  - Switched-Linear, Uncertain
- Constraints
  - States and Inputs

Solved Problem ~ 40 years ago

- Objective
  - Minimization of performance index
- Models
  - Linear, Uncertain
  - Switched-Linear, Uncertain
- Constraints
  - States and Inputs

Focus of Research ~ 10 years ago

- Objective
  - Minimization of performance index
- Models
  - Linear, Uncertain
  - Switched-Linear, Uncertain
- Constraints
  - States and Inputs

Balluchi, Bemporad, Di Benedetto, Goodwin, Johansen, Johansson, Kerrigan, Maciejowski, Mayne, Morari, Pappas, Rantzer, Rawlings, Sangiovanni-Vincentelli, Sastry, Sontag, Tomlin**,** …. **and many others.**

Hybrid Constrained Optimal Control

Borrelli from 1999 to 2004

$$\min_U \sum_{k=0}^{N} ||Qx(k)||_p + ||Ru(k)||_p$$

subj.to

$$x(k+1) = A_i x(k) + B_i u(k) + f_i \quad \text{if } [x(k), u(k)] \in \mathcal{X}_i, \ i = 1, \ldots, s$$

$$Ex(k) + Lu(k) \leq M, \quad k = 0, 1, 2, \ldots$$

$$x(k) \in \mathrm{R}^n \times \{0, 1\}^{n_b}, \quad u(k) \in \mathrm{R}^m \times \{0, 1\}^{m_b}, \quad U \triangleq \{u(0), u(1), u(2), \ldots\}$$

- Understanding solution structures and properties
- Solution computational methods and tools

Hybrid Constrained Optimal Control

Borrelli from 1999 to 2004

$$\min_U \quad \sum_{k=0}^{N} ||Qx(k)||_p + ||Ru(k)||_p$$

$$\text{subj.to} \quad x(k+1) = A_i x(k) + B_i u(k) + f_i \quad \text{if } [x(k), u(k)] \in \mathcal{X}_i, \quad i = 1, \ldots, s$$

$$Ex(k) + Lu(k) \leq M, \quad k = 0, 1, 2, \ldots$$

$$x(k) \in \mathrm{R}^n \times \{0,1\}^{n_b}, \quad u(k) \in \mathrm{R}^m \times \{0,1\}^{m_b}, \quad U \triangleq \{u(0), u(1), u(2), \ldots\}$$

- **Understanding solution structures and properties**

- Solution computational methods and tools

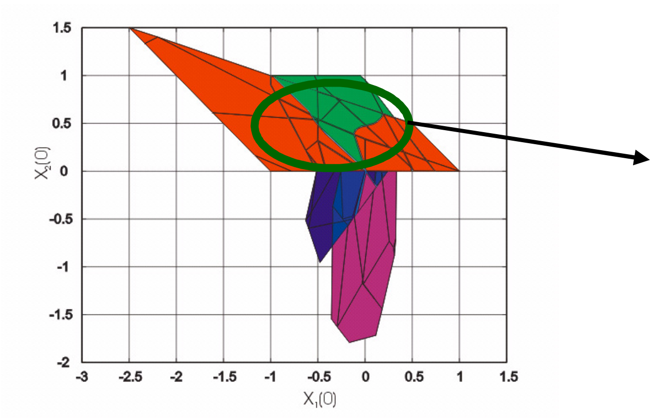
Characterization of the Solution (p=1,2,∞)

Borrelli et al, ACC, 2000
Borrelli et al, AUTOMATICA, 2005

The solution to the optimal control problem is a time varying PWA state feedback control law of the form

$$u^*(k)(x) = \left\{ \begin{array}{lll} F_1(k)x + G_1(k) & \text{if} & x \in CR_1(k) \\ \vdots & \vdots & \vdots \\ F_R(k)x + G_R(k) & \text{if} & x \in CR_R(k) \end{array} \right\}$$

$\{CR_i\}_{i=1}^R$ is a partition of the set of feasible states $x(k)$.



- **p=1, p=∞:**

  $CR_i(k) \triangleq \{x : M_i(j,k)x \leq K_i(j,k)\}$

- **p=2:**

  $CR_i(k) \triangleq \{x : x'L_i(j,k)x + M_i(j,k)x \leq K_i(j,k)\}$

Hybrid Constrained Optimal Control

Borrelli from 1999 to 2004

$$\min_U \quad \sum_{k=0}^{N} ||Qx(k)||_p + ||Ru(k)||_p$$

$$\text{subj.to} \quad x(k+1) = A_i x(k) + B_i u(k) + f_i \quad \text{if } [x(k), u(k)] \in \mathcal{X}_i, \quad i = 1, \ldots, s$$

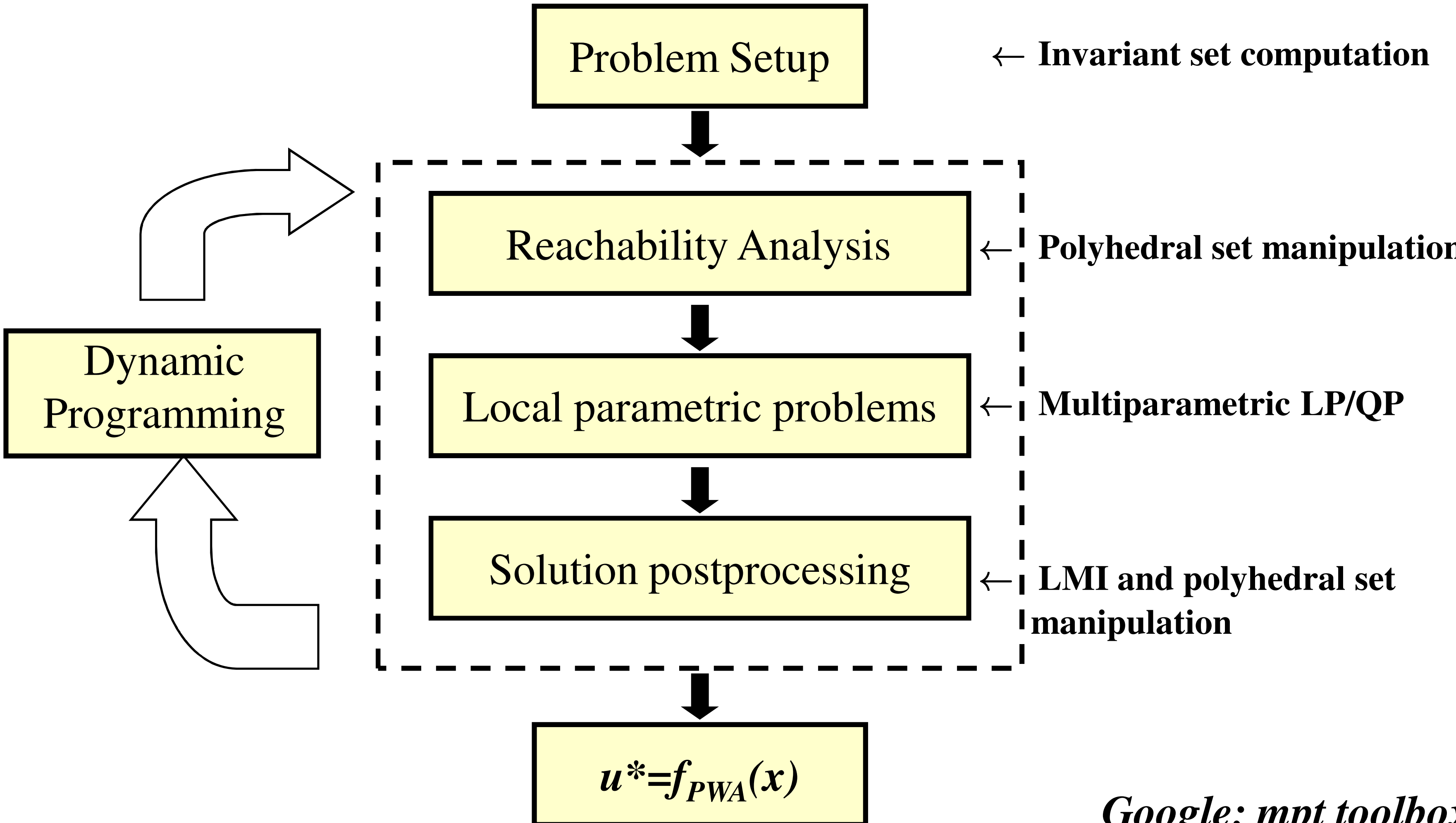
$$Ex(k) + Lu(k) \leq M, \quad k = 0, 1, 2, \ldots$$

$$x(k) \in \mathrm{R}^n \times \{0,1\}^{n_b}, \quad u(k) \in \mathrm{R}^m \times \{0,1\}^{m_b}, \quad U \triangleq \{u(0), u(1), u(2), \ldots\}$$

- Understanding solution structures and properties
- **Solution computational methods and tools**

Computational Flow

Borrelli et al, JOTA, 2003
Borrelli et al, AUTOMATICA, 2006
Baotic, Borrelli et al, SICON, 2007



Google: mpt toolbox

Summary

Systematic Model-Based Control Design
MIMO, PWA, Constraints, Logics

- Piecewise affine state feedback control law
- Off-line computation:
  Automatic partitioning and control law synthesis
- On-line computation: Lookup Table Evaluation
- Extended to Min-Max Constrained Problems

Borrelli, Bemporad, Morari, TAC, 2003

MPC Algorithm

$$\min_U J(U, x(0)) \triangleq \sum_{k=0}^{N-1} \|Q(x(k) - x_{ref})\|_p + \|R(u(k) - u_{ref})\|_p$$

$$\text{subj. to} \begin{cases} x(k+1) = f(x(k), u(k)) \\ u(k) \in \mathcal{U} \\ x(k) \in \mathcal{X} \\ x(0) = x(t) \end{cases}$$

At time t:

- Measure (or estimate) the current state x(t)
- Find the optimal input sequence $U^* \triangleq \{u^*(t), u^*(t+1), \ldots, u^*(t+N)\}$
- Apply only u(t)=u*(t) , and discard *u*(t+1), u*(t+2)*, …

Repeat the same procedure at time *t +1*

Important Issues in Model Predictive Control

Even assuming perfect model, no disturbances:

predicted open-loop trajectories $\neq$ closed-loop trajectories

- **Feasibility**
  Optimization problem may become infeasible at some future time step.

- **Stability**
  Closed-loop stability is not guaranteed.

- **Performance**
  Goal: $\min \sum_{i=0}^{\infty} L(x(k+i), u(k+i))$
  What is achieved by repeatedly minimizing $\sum_{i=0}^{N-1} L(x(k+i), u(k+i))$

Feasibility and Stability Constraints

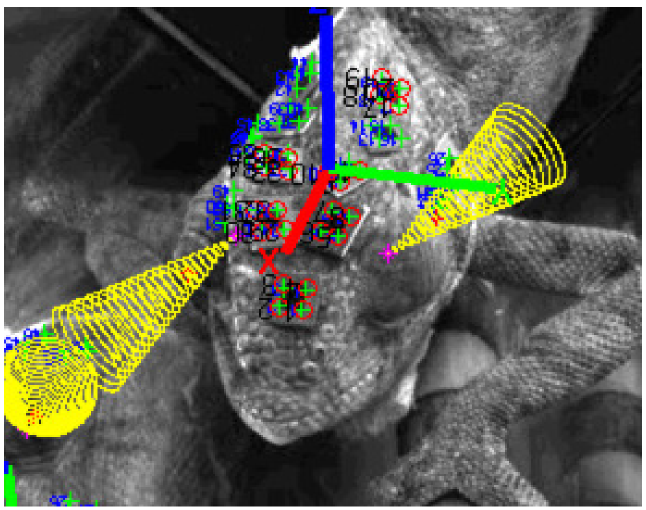
$$\min_U J(U, x(0)) \triangleq P(x(N)) + \sum_{k=0}^{N-1} \|Qx(k)\|_p + \|Ru(k)\|_p$$

$$\text{subj. to} \begin{cases} x(k+1) = f(x(k), u(k)) \\ u(k) \in \mathcal{U} \\ x(k) \in \mathcal{X} \\ x(0) = x(t) \\ x(N) \in \mathcal{X}_f \end{cases}$$

X_f is an **Invariant Set**

P(x) *is a* **Control Lyapunov Function**.

Chameleon Visual Tracking



Objective

Tracking of a moving prey

Model

PTZ camera: Linear

Prey: Linear point mass

Constraints

Pan Tilt and Zoom constraints

Prey in tracking window ∀ unknown bounded accelerations

Min-Max Predictive Control

$$J_j^*(x_j) = \min_{u_j} J_j(x_j, u_j)$$

$$\text{subj. to} \begin{cases} \text{Model} \\ \text{Constaints} \end{cases}$$

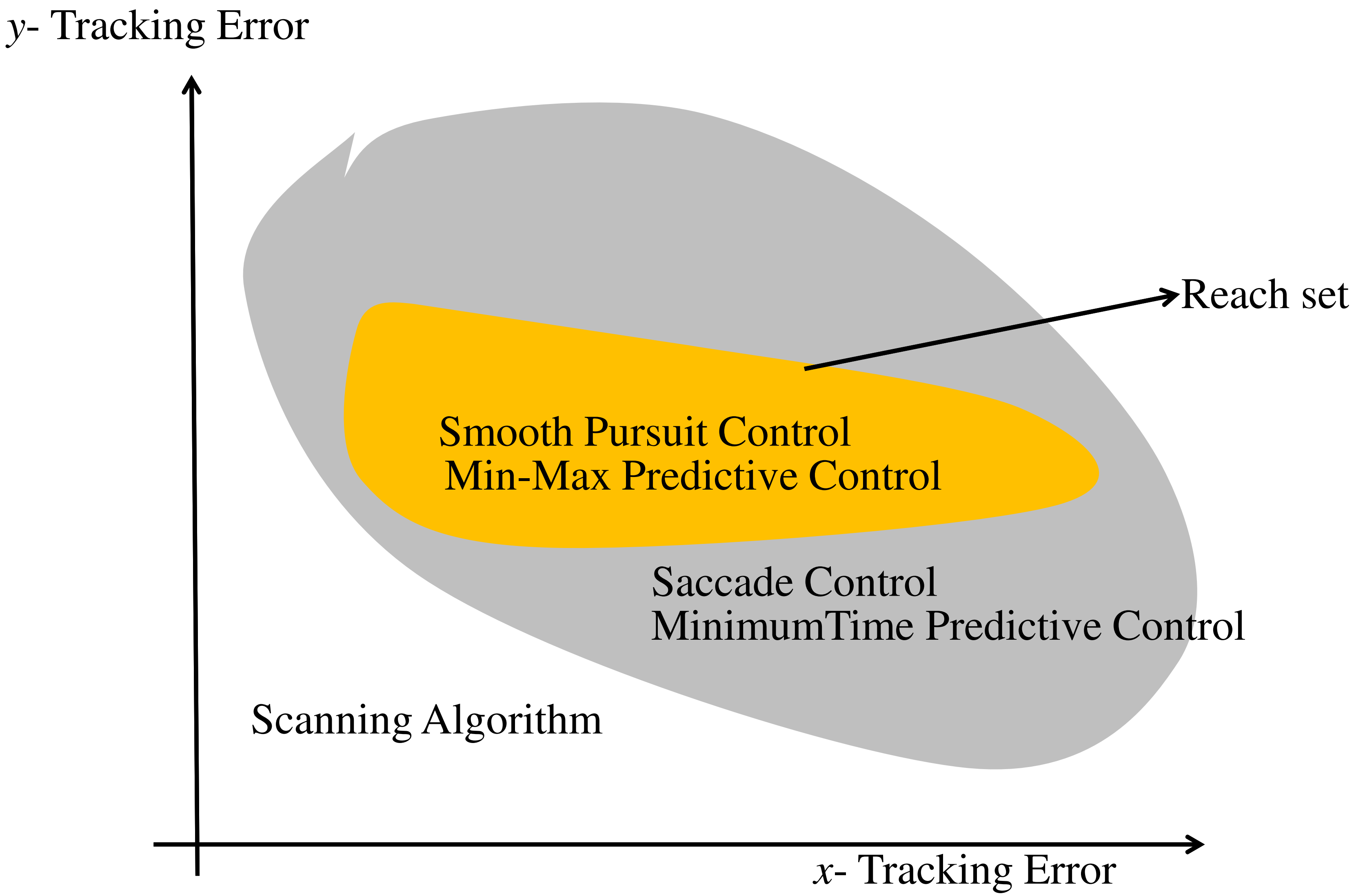
$$J_j(x_j, u_j) = \max_{v_j, w_j} \left(\|Qx_j\|_p + \|Ru_j\|_p + V^*(x_{j+1}) \right)$$

Model: $x_{j+1} = A(w_j)x_j + B(w_j)u_j + Ev_j$

Uncertainty: Additive $v_i \in \mathcal{V}$, Polytypic $w_i \in \mathcal{W}$

Constraints: $Fx_j + Gu_j \leq f$ *For all* $v_i \in \mathcal{V}$, $w_i \in \mathcal{W}$

Addressing Feasibility: Control Law Design



Robotic Chameleon Video

Avin, Borrelli et al., IROS, 2006

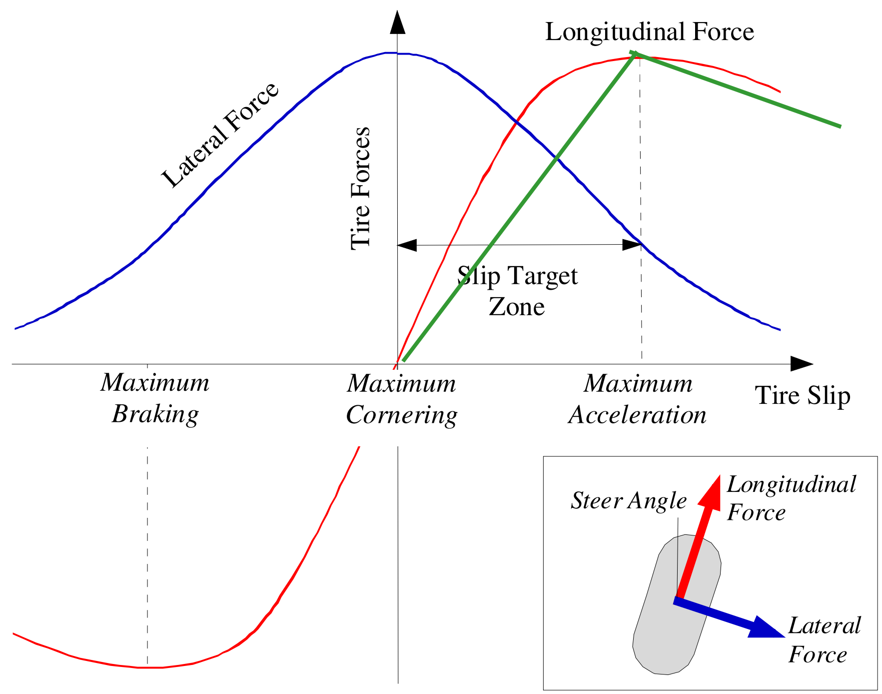
Explicit Min-Max MPC Solved at 50Hz

Vehicle Dynamics Control

A driver aid for atypical road conditions, such as slippery, windy an bumpy roads

- ✓ Traction Control (TC)
- ✓ Anti-lock Braking System (ABS)
- ✓ Electronic Stabilty Program (ESP)

- ✓ Active Front Steering (AFS) systems
- ✓ Active Suspension systems
- ✓ Active differential systems



Nonlinear (Piece-wise linear) and Constrained System

Traction Control Experiment

2000 Ford Focus, 2.0l 4-cyl Engine, 5-speed Manual Trans

Borrelli et al., IEEE TCST, 2006

Ford Fusion Production Controller

"..Traction control on the V-6 test car was just right -- perhaps unique in all the industry…."

USA Today (Oct. 28-2005)

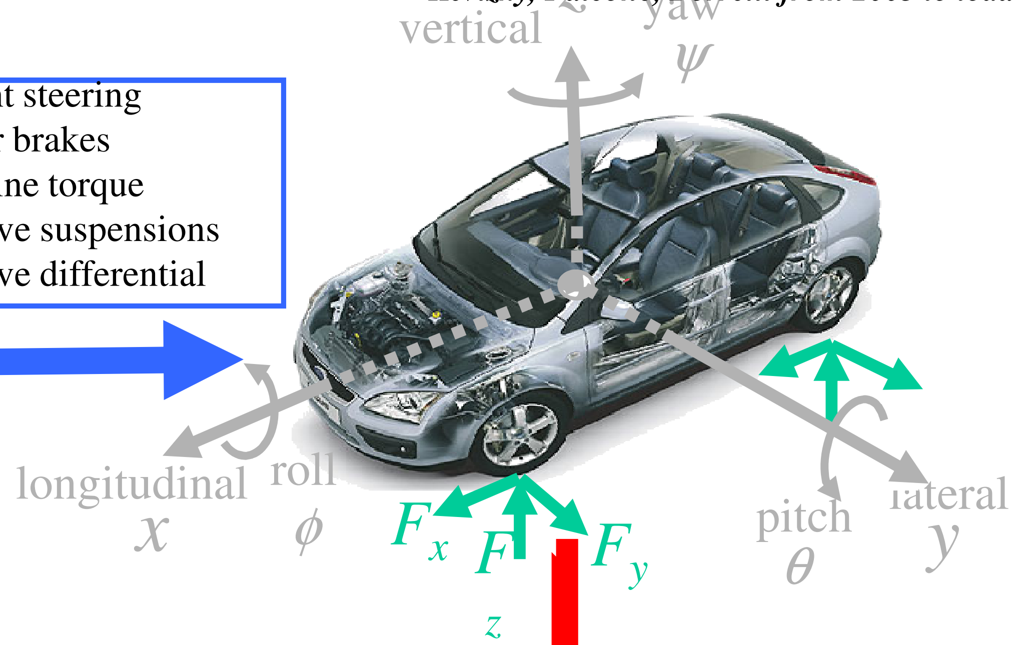
Ford Motor Company

Integrated VDC via MPC

Kevizky, Falcone, Borrelli from 2003 to today

MIMO controller integrating local and global measurements coming from GPS, cameras, infrared and radar

- ✓ Front steering
- ✓ Four brakes
- ✓ Engine torque
- ✓ Active suspensions
- ✓ Active differential



- ✓ . Longitudinal, lateral and vertical velocity/accelerations
- ✓ Yaw, roll and pitch angles/rates
- ✓ Position and velocity in a global frame

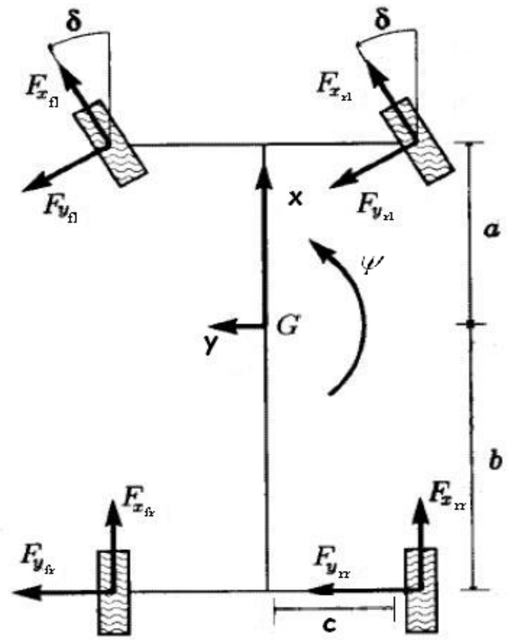
Controlling Yaw, Roll, Pitch, Vertical, Lateral and Longitudinal Dynamics via Multiple Input

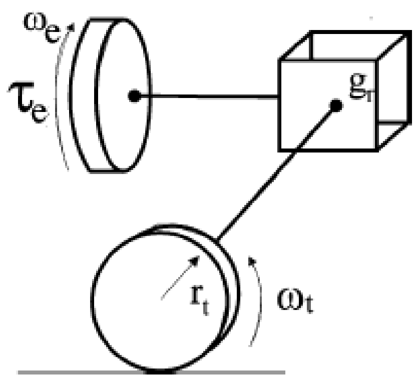
Enabling path following capabilities

Davor Hrovat, Jahan Asgari, Eric Tseng, Mike Fodor

Ford Motor Company

Vehicle Model - 11 States, 6 Inputs





Inputs

δ_f Front steering angle

F_b FL,FR,RL,RR brakes

τ Desired engine torque

States

$\dot{y}$ Lateral velocity

$\dot{x}$ Longitudinal velocity

ψ Yaw angle

$\dot{\psi}$ Yaw rate

Y Lateral position (I.F.)

X Longitudinal position (I.F.)

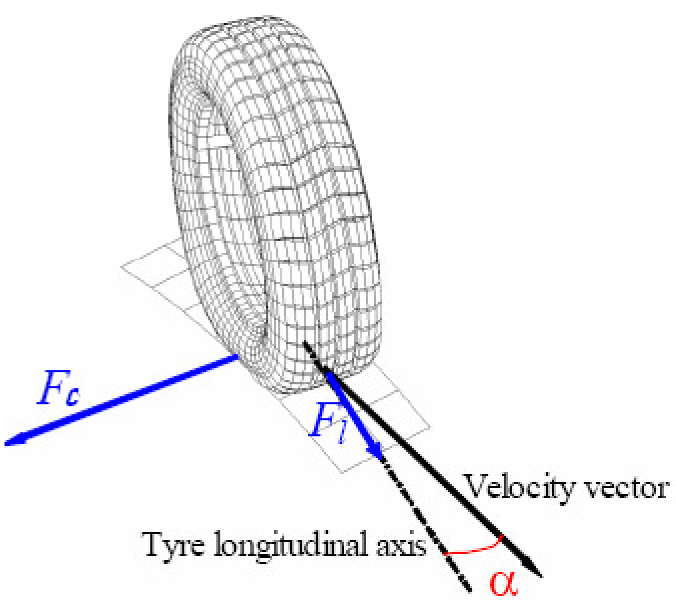
$$x = [y, \dot{y}, \dot{x}, \psi, \dot{\psi}, Y, X, \omega_{fl}, \omega_{fr}, \omega_{rl}, \omega_{rr}]$$

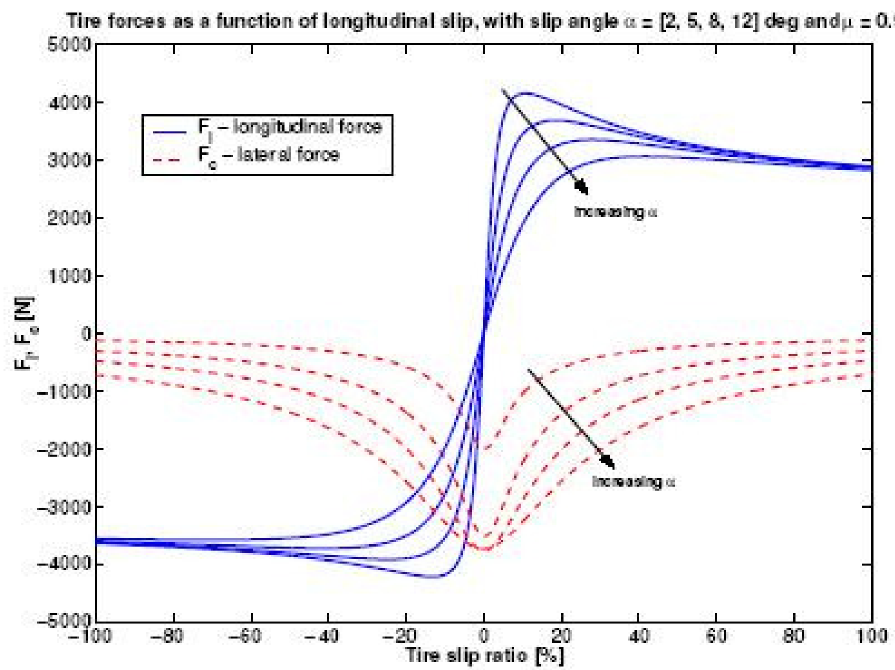
Ford Motor Company

Pacejka Tire model

$$F = f(\alpha, s, \mu, F_z)$$

Semi-empirical model calibrated on experimental data





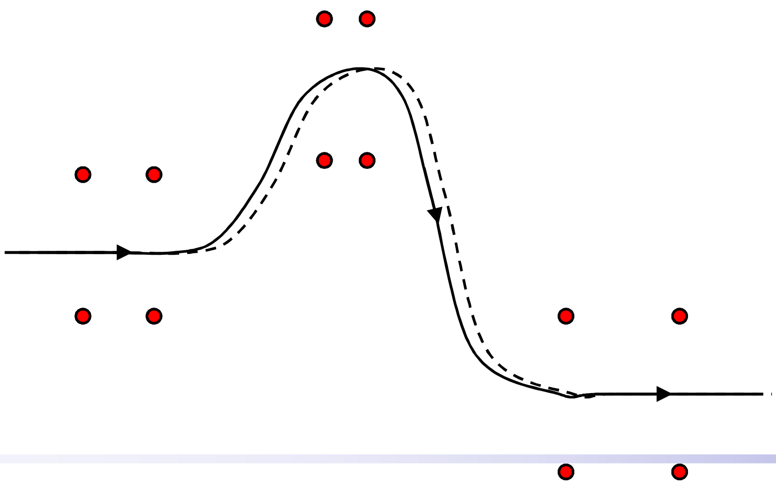
Autonomous Vehicle
Tests and Experimental setup

Objective

- Minimize angle and lateral distance deviations from reference trajectory
- Double lane change
- Driving on snow/ice, at different entry speeds

System

- Jaguar X-type
- dSpace rapid prototyping system equipped with a DS1005 processor board Sampling time: 50 ms
- Differential GPS, gyros, lateral accelerometers



Acknowledgements

- **University of Minnesota (Minneapolis, USA)**
  - Tamas Keviczky, Gary Balas

- **Unisannio (Benevento, Italy)**
  - Paolo Falcone

- **Honeywell Labs (Minneapolis,USA)**
  - Dharmashankar Subramanian, Kingsley Fregene, Datta Godbole
  - Sonja Glavaski, Greg Stewart, Tariq Samad

- **Ford Research Labs (Dearborn,USA)**
  - Jahan Asgari, Eric Tseng, Davor Hrovat

- **Technion (Haifa, Israel)**
  - Ofiv Avni, Gadi Katzirinst, Ehud Rivlin, Hector Rotstein

- **ETH (Zurich, Switzerland)**
  - Mato Baotic, Alberto Bemporad, Manfred Morari