A Switching Control Approach to Stabilization of Parameter Varying Time Delay Systems

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OUTLINE

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Problem Motivation

Consider a simple finite dimensional linear system

$$\dot{x}(t) = A_k x(t) \ , \quad k \in \{1, \ldots, \ell\}$$

$A_1, \ldots, A_\ell$ are known fixed matrices

Assume each fixed "candidate" system is stable, i.e.,

$$\text{max real } (\text{eig}(A_k)) < 0 \ , \qquad \forall \ k$$

It is well known that this is not sufficient to guarantee asymptotic stability of the switched system. In other words, there may exist a switching sequence leading to an unstable system.

There are *dwell time based stability results*: if the minimum time interval between consecutive switchings is longer than a certain dwell time, then the system is asymptotically stable.

This paper deals with LPV systems with time delays in this context.

LPV Systems with Time Delays

Consider the following linear parameter varying time delay systems:

$$\Sigma_\theta : \begin{cases} \dot{x}(t) & = & A(\theta)x(t) + \bar{A}(\theta)x(t-\tau(\theta)) + B(\theta)u(t), \quad t \geq 0 \\ x_0(\xi) & = & \phi(\xi), \quad \forall \xi \in [-\tau_{max}, 0] \end{cases}$$

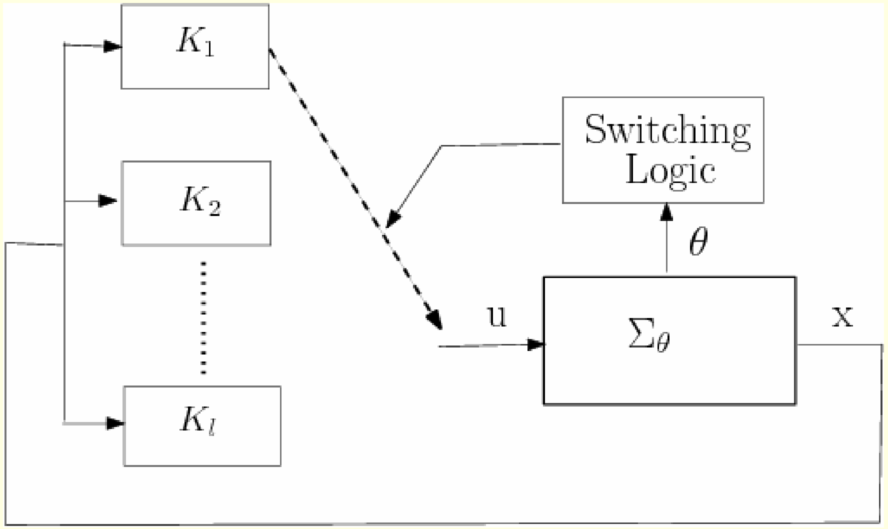
where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is control input, $\tau(\theta)$ denotes the parameter varying time-delay satisfying $0 < \tau(\theta) \leq \tau_{max}$. The LPV time delay system Σ_θ depends on a parameter $\theta(t)$, where $\theta(t) \in \mathbb{R}$ is assumed to be continuously differentiable and $\theta \in \Theta$ where Θ is a compact set.

We propose to construct a family of stabilizers designed at selected operating points

$\mathcal{K} := \{K_i : i = 1, 2, ..., l\}$, where K_i is a state feedback controller designed for $\theta = \theta_i$, which robustly stabilizes the LPV time delay systems for

$$\theta_i \in \Theta_i := [\theta_i^-, \theta_i^+], \qquad \Theta \subseteq \bigcup_{i=1}^{l} \Theta_i.$$

Switched feedback system:



The switched feedback system satisfy

$$\Sigma_q : \begin{cases} \dot{x}(t) &= A^c_{q(t)}(\theta)x(t) + \bar{A}(\theta)x(t-\tau(\theta)), \quad t \geq 0 \\ x_0(\xi) &= \phi(\xi), \quad \forall \xi \in [-\tau_{max}, 0] \end{cases}$$

where $q(t)$ is a piecewise switching signal taking values on the set $\mathcal{F} := \{1, 2, ..., \ell\}$, i.e. $q(t) = k_j$, $k_j \in \mathcal{F}$, for $\forall t \in [t_j, t_{j+1})$, where t_j, $j \in \mathbb{Z}^+ \cup \{0\}$, is the j^{th} switching time instant. We also assume $u(t) = K_{k_j}x(t)$ for $\theta \in \Theta_{k_j}$.

Feedback system with uncertainty:

In $t \in [t_j, t_{j+1})$ we define $\tau_{k_j}(\theta) := \tau(\theta)$, for $\theta \in \Theta_{k_j}$

$$
\begin{aligned}
A(\theta) &= A_{k_j} + \Delta A(\theta), \text{ where } \Delta A(\theta) := D_{k_j} F_{k_j}(\theta) E_{k_j}, \\
\bar{A}(\theta) &= \bar{A}_{k_j} + \Delta \bar{A}(\theta), \text{ where } \Delta \bar{A}(\theta) := \bar{D}_{k_j} \bar{F}_{k_j}(\theta) \bar{E}_{k_j}, \\
B(\theta) &= B_{k_j} + \Delta B(\theta), \text{ where } \Delta B(\theta) := D_{k_j} F_{k_j}(\theta) E^B_{k_j}
\end{aligned}
$$

$$
F_{k_j}(\theta)^T F_{k_j}(\theta) \leq I \qquad \text{and} \qquad \bar{F}_{k_j}(\theta)^T \bar{F}_{k_j}(\theta) \leq I
$$

$$
\Sigma_{k_j} : \begin{cases} \dot{x}(t) = (A^c_{k_j} + \Delta A^c_{k_j}(\theta))x(t) + (\bar{A}_{k_j} + \Delta \bar{A}(\theta))x(t - \tau_{k_j}(\theta)), \quad t \in [t_j,\ t_{j+1}) \\ x_{t_j}(\xi) = \phi_j(\xi), \quad \forall \xi \in [-\bar{\tau}_{k_j}, 0], \end{cases}
$$

$$
\begin{aligned}
A^c_{k_j} &= A_{k_j} + B_{k_j} K_{k_j}, \\
\Delta A^c_{k_j} &= D_{k_j} F_{k_j}(\theta) E^c_{k_j}, \\
E^c_{k_j} &= E_{k_j} + E^B_{k_j} K_{k_j}
\end{aligned}
$$

$$
\phi_j(\xi) = \begin{cases} x(t_j + \xi) & -\bar{\tau}_{k_j} \leq \xi < 0 \\ \lim_{h \to 0^-} x(t_j + h), & \xi = 0 \end{cases} \qquad 0 < \bar{\tau}_{k_j} := \max \tau_{k_j}(\theta), \text{for } \theta \in \Theta_{k_j},
$$

Definition of asymptotic stability:

We say the feedback system is ***stable*** if there exists a function ϑ satisfying

$\|x(t)\| \leq \vartheta(|x|_{[t_0-\tau_{max},t_0]}), \quad \forall t \geq t_0 \geq 0,$ along the trajectory of Σ_q

and $\vartheta(\cdot): \mathbb{R}^+ \to \mathbb{R}^+$ is strictly increasing with $\vartheta(0)=0$ furthermore,

the system is ***asymptotically stable*** if it is stable and $\lim_{t\to+\infty} x(t) = 0$

We will first consider the stability of each candidate system defined earlier:

$$\Sigma_{k_j}: \begin{cases} \dot{x}(t) &= (A^c_{k_j} + \Delta A^c_{k_j}(\theta))x(t) + (\bar{A}_{k_j} + \Delta\bar{A}(\theta))x(t-\tau_{k_j}(\theta)), \quad t \in [t_j,\, t_{j+1}) \\ x_{t_j}(\xi) &= \phi_j(\xi), \quad \forall \xi \in [-\bar{\tau}_{k_j}, 0], \end{cases}$$

For this purpose we will look for a Lyapunov-Razumikhin function

$V_{k_j}(t, x_j) = x_j^T(t) P_{k_j} x_j(t), \quad t \in [t_j, t_{j+1}]$ so that

$$\kappa_{k_j}\|x_j(t)\|^2 \leq V_{k_j}(t,x_j) \leq \bar{\kappa}_{k_j}\|x_j(t)\|^2, \ \forall x_j \in \mathbb{R}^n,$$

$$\kappa_{k_j} := \sigma_{min}[P_{k_j}] > 0 \qquad \bar{\kappa}_{k_j} := \sigma_{max}[P_{k_j}] > 0$$

First assume for some constant $p_{k_j} > 1$ we have

$$x_j^T(t+\varphi)P_{k_j}x_j(t+\varphi) < p_{k_j}x_j^T(t)P_{k_j}x_j(t) \quad \forall\varphi \in [-2\bar{\tau}_{k_j}, 0].$$

Moreover, if P_{k_j} is chosen to satisfy

$$\begin{array}{rcl} ((A^c_{k_j} + \Delta A^c_{k_j})^T Q^{-1}_{k_j}((A^c_{k_j} + \Delta A^c_{k_j}) & \leq & P_{k_j} \\ (\bar{A}_{k_j} + \Delta\bar{A})^T \bar{Q}^{-1}_{k_j}(\bar{A}_{k_j} + \Delta\bar{A}) & \leq & P_{k_j} \end{array}$$

for some matrices $Q_{k_j} > 0, \bar{Q}_{k_j} > 0$ then we have

$$\dot{V}_{k_j}(t, x_j) \leq -x_j^T(t)S_{k_j}x_j(t),$$

where

$$\begin{array}{rcl} S_{k_j} := & - & \{P_{k_j}(A_{k_j} + B_{k_j}K_{k_j} + \bar{A}_{k_j}) + (A_{k_j} + B_{k_j}K_{k_j} + \bar{A}_{k_j})^T P_{k_j} \\ & + & \gamma_{k_j}P_{k_j}D_{k_j}D^T_{k_j}P_{k_j} + \gamma^{-1}_{k_j}(E_{k_j} + E^B_{k_j}K_{k_j})^T(E_{k_j} + E^B_{k_j}K_{k_j}) \\ & + & \bar{\gamma}_{k_j}P_{k_j}\bar{D}_{k_j}\bar{D}^T_{k_j}P_{k_j} + \bar{\gamma}^{-1}_{k_j}\bar{E}^T_{k_j}\bar{E}_{k_j} + 2\bar{\tau}_{k_j}p_{k_j}P_{k_j} \\ & + & \bar{\tau}_{k_j}P_{k_j}\bar{A}_{k_j}(Q_{k_j} + \bar{Q}_{k_j})\bar{E}^T_{k_j}(\epsilon_{k_j}I - \bar{E}_{k_j}(Q_{k_j} + \bar{Q}_{k_j})\bar{E}^T_{k_j})^{-1}\bar{E}_{k_j}(Q_{k_j} + \bar{Q}_{k_j})\bar{A}^T_{k_j}P_{k_j} \\ & + & \bar{\tau}_{k_j}P_{k_j}(\bar{A}_{k_j}(Q_{k_j} + \bar{Q}_{k_j})\bar{A}^T_{k_j} + \epsilon_{k_j}\bar{D}_{k_j}\bar{D}^T_{k_j})P_{k_j}, \end{array}$$

where $\gamma_{k_j} > 0$, $\bar{\gamma}_{k_j} > 0$, $\epsilon_{k_j} > 0$ are free design parameters.

The goal is to find such positive matrices and scalars, and the control gain so that $S_{k_j} > 0$ which implies

$$\dot{V}_{k_j}(t, x_j) < -w_{k_j}\|x_j\|^2 \quad \text{where} \quad w_{k_j} := \sigma_{min}[S_{k_j}] > 0.$$

Lemma. The time varying time delay system given above is robustly stable if there exist matrices $Y_{k_j}, X_{k_j} > 0,\ Q_{k_j} > 0,\ \bar{Q}_{k_j} > 0,$ and scalars $p_{k_j} > 1,\ \gamma_{k_j} > 0,\ \bar{\gamma}_{k_j} > 0,\ \epsilon_{k_j} > 0,\ \rho_{k_j} > 0,\ \bar{\rho}_{k_j} > 0,$ such that

$$\begin{bmatrix} X_{k_j} & X_{k_j}A_{k_j}^T + Y_{k_j}^T B_{k_j}^T & X_{k_j}E_{k_j}^T + Y_{k_j}^T (E_{k_j}^B)^T \\ A_{k_j}X_{k_j} + B_{k_j}Y_{k_j} & Q_{k_j} - \rho_{k_j} D_{k_j} D_{k_j}^T & 0 \\ E_{k_j}X_{k_j} + E_{k_j}^B Y_{k_j} & 0 & \rho_{k_j} I \end{bmatrix} \geq 0$$

$$\begin{bmatrix} X_{k_j} & X_{k_j}\bar{A}_{k_j}^T & X_{k_j}\bar{E}_{k_j}^T \\ \bar{A}_{k_j}X_{k_j} & \bar{Q}_{k_j} - \bar{\rho}_{k_j}\bar{D}_{k_j}\bar{D}_{k_j}^T & 0 \\ \bar{E}_{k_j}X_{k_j} & 0 & \bar{\rho}_{k_j} I \end{bmatrix} \geq 0$$

$$\begin{bmatrix} M_{k_j} & X_{k_j}E_{k_j}^T + Y_{k_j}^T (E_{k_j}^B)^T & X_{k_j}\bar{E}_{k_j}^T & \bar{\tau}_{k_j}\bar{A}_{k_j}(Q_{k_j} + \bar{Q}_{k_j})\bar{E}_{k_j}^T \\ E_{k_j}X_{k_j} + E_{k_j}^B Y_{k_j} & \gamma_{k_j} I & 0 & 0 \\ \bar{E}_{k_j}X_{k_j} & 0 & \bar{\gamma}_{k_j} I & 0 \\ \bar{\tau}_{k_j}\bar{E}_{k_j}(Q_{k_j} + \bar{Q}_{k_j})^T \bar{A}_{k_j}^T & 0 & 0 & -\bar{\tau}_{k_j}(\epsilon_{k_j} I - \bar{E}_{k_j}(Q_{k_j} + \bar{Q}_{k_j})\bar{E}_{k_j}^T) \end{bmatrix} < 0$$

$$\begin{aligned} M_{k_j} &= (A_{k_j} + \bar{A}_{k_j})X_{k_j} + X_{k_j}(A_{k_j} + \bar{A}_{k_j})^T + \gamma_{k_j} D_{k_j} D_{k_j}^T + \bar{\gamma}_{k_j}\bar{D}_{k_j}\bar{D}_{k_j}^T + B_{k_j}Y_{k_j} + Y_{k_j}^T B_{k_j}^T \\ &\quad + \bar{\tau}_{k_j}\epsilon_{k_j}\bar{D}_{k_j}\bar{D}_{k_j}^T + \bar{\tau}_{k_j}\bar{A}_{k_j}(Q_{k_j} + \bar{Q}_{k_j})\bar{A}_{k_j}^T + 2\bar{\tau}_{k_j} p_{k_j} X_{k_j}. \end{aligned}$$

In this case a stabilizing state feedback controller is given by $K_{k_j} = Y_{k_j} X_{k_j}^{-1}$, with $P_{k_j} = X_{k_j}^{-1}$.

Theorem: Consider the switched LPV time delay system with state feedback controllers designed as above. Let the dwell time be defined by

$$\tau_D := T^* + 2\tau_{max}, \quad \text{where} \quad T^* := \lambda\mu\left\lfloor \frac{\lambda - 1}{\bar{p} - 1} + 1 \right\rfloor$$

$$\bar{p} := \min_{k_j \in \mathcal{F}} \{p_{k_j}\} > 1 \quad \text{and} \quad \lfloor \cdot \rfloor \quad \text{is the floor integer function}$$

$$\lambda := \max_{k_j \in \mathcal{F}} \frac{\bar{\kappa}_{k_j}}{\kappa_{k_j}}, \quad \text{and} \quad \mu := \max_{k_j \in \mathcal{F}} \frac{\bar{\kappa}_{k_j}}{w_{k_j}}.$$

Then system is asymptotically stable for any switching rule, provided that the minimum time interval between consecutive switchings is greater than τ_D.

Recall:

$$\kappa_{k_j} := \sigma_{min}[P_{k_j}] > 0 \qquad \bar{\kappa}_{k_j} := \sigma_{max}[P_{k_j}] > 0$$

$$w_{k_j} := \sigma_{min}[S_{k_j}] > 0$$

maximize ***minimize***

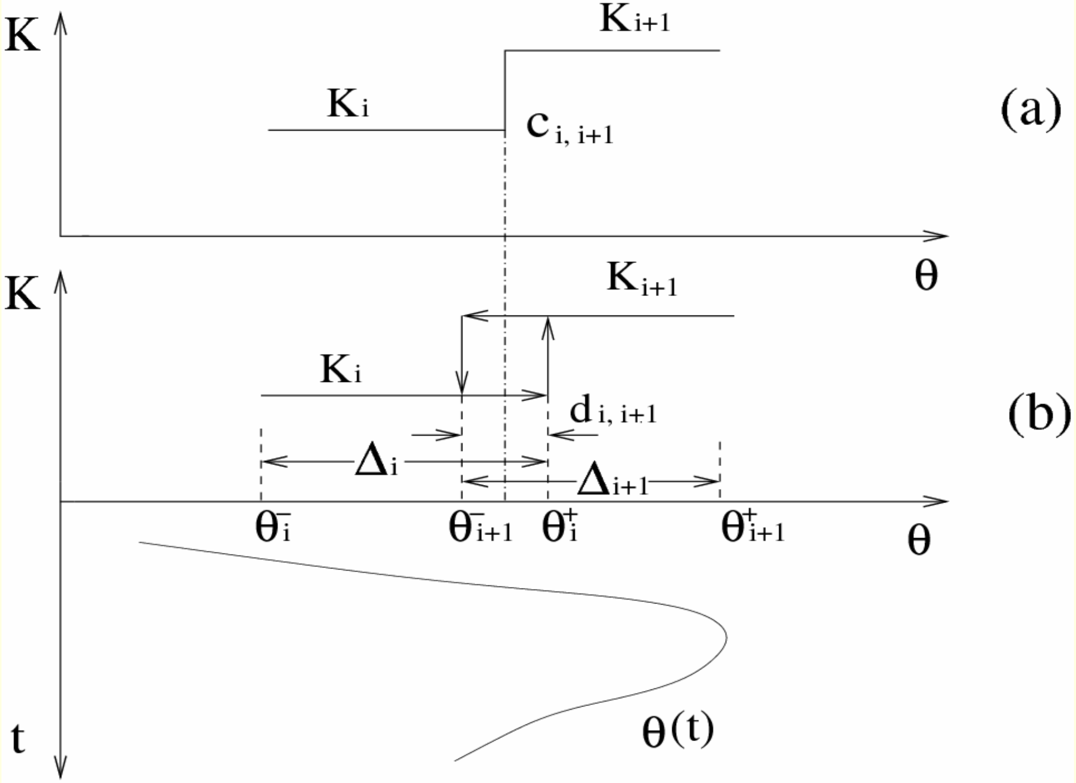
Two Switching Mechanisms:

(a) Midpoint switching:

dwell time based stability results do not work in this case.

(b) Hysterisis switching:

if parameter variation is slow stability can be guaranteed.

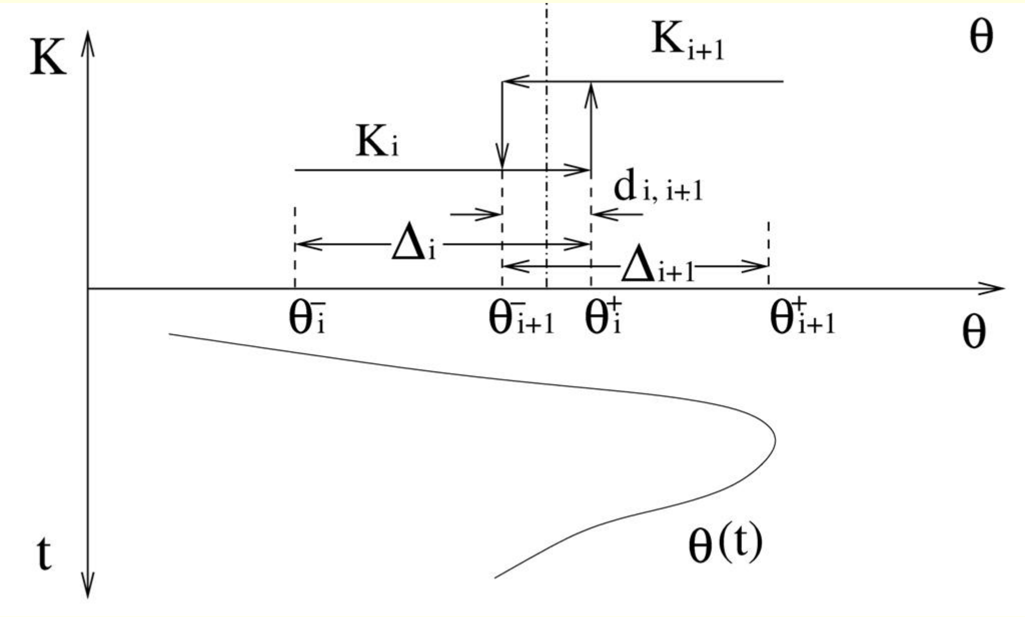


Corollary:

For the hysterisis switching a sufficient condition for asymptotic stability is

$$|\dot{\theta}(t)| < \min_{i \in \mathcal{F}} \left\{ \frac{|d_{i,i+1}|}{\tau_D} \right\}, \text{ where}$$

$d_{i,i+1} = \Theta_i \cap \Theta_{i+1}$ is the i^{th} hysterisis interval as shown in in the figure.



Concluding Remarks

Dwell time based stability result is obtained for switched LPV time delay systems in the form

$$\dot{x}(t) = A(\theta)x(t) + \bar{A}(\theta)x(t-\tau(\theta)) + B(\theta)u(t)$$

$$u(t) = Kx(t)$$

Feedback gain is designed using standard techniques similar to published papers in the *robust control of time delay systems* literature.

As a future work it would be interesting to study the case where there is a time delay in the feedback loop.

Another interesting future study is to consider the output feedback problem.

Solving the BMIs is a difficult problem in general.

- Fixing some of the scalar parameters make them LMIs.
- Trying to minimize maximum singular values and maximize the minimum singular values of the solutions of these LMIs will help minimizing the dwell time, that will allow faster switchings.
- In the full version of the paper we will include an example.