

FUZZY L1 NORM STRATEGY FOR MODEL BASED CONTROL

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INTRODUCTION

The formulation of Model Based control (MBC) depends heavily on the quality of the model chosen. It is therefore of greatest importance to select a model structure and a set of model parameters where there are a great deal of uncertainty as well as vague phenomena to obtain a model with sufficient predictive precision. So, this presentation considers the parameter estimation problem of MBC model formulated based on fuzzy linear programming which utilizes fuzzy parameters consisting of an ordered pair (α, c) which describes the center and width of the fuzzy parameters respectively and suggests the control strategy to the control of stable and unstable systems.

Fuzzy linear regression (FLR), was proposed by Tanaka and coworkers in 1982. Many different fuzzy regression approaches have been proposed by different investigators since then [22-24]The Tanaka and Kandel[18,25] approach is still one of the most frequently used analysis due to its use of linear programming and its simplicity.

FUZZY ARX MODEL PARAMETERIZATION BY FUZZY LINEAR REGRESSION

$$y(k) = \sum_{i=1}^{na} a_i y(k-i) + \sum_{i=1}^{nb} b_i u(k-\delta-i+1) \quad (1)$$

The Auto-Regression model with eXogenous input or ARX model for identification has been considered as in Eq(1). And then This model was built using fuzzy linear regression techniques and non-fuzzy output data.

In our approach we develop a parameterization of the plant that is affine in the unknown parameters. The "computed variable" \hat{y} is given by the model.

$$\hat{y} = A_1 \varphi_1(x) + A_2 \varphi_2(x) + \dots + A_p \varphi_p(x) \quad p = 1, 2, \dots, n_a + n_b \quad (2)$$

Where \hat{y} is the "computed or estimated value of the variable yi

$\varphi_1, \varphi_2, \dots, \varphi_p$ are known functions of states (x)

A_1, A_2, \dots, A_p are the p number of unknown parameters of the model to be determined.

In this study, A_p was assumed to be a symmetric triangular fuzzy number with center α_p and half-width, c_p , $c_p \geq 0$

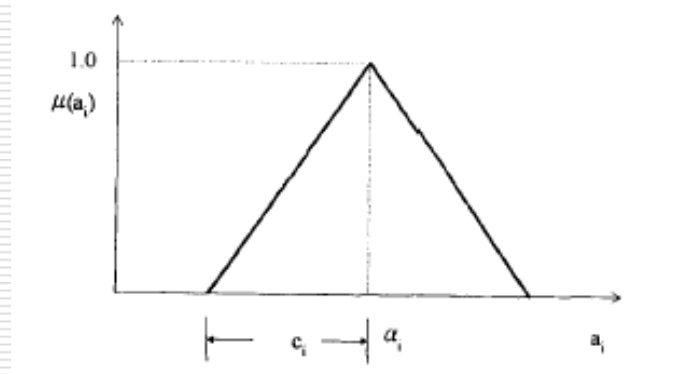


Fig.1:Fuzzy Parameter A

$$\mu_{A_j}(a_j) = \begin{cases} 1 - \frac{|\alpha_j - a_j|}{c_j}, & \alpha_j - c_j \leq a_j \leq \alpha_j + c_j \\ 0 & \text{otherwise} \end{cases}$$

where A is a fuzzy set on the product space of parameters whose membership function is denoted by $\mu_A(a)$.

Definition 1 (Tanaka et al. [25]). The fuzzy function is denoted by

$$f: X \rightarrow \mathcal{F}(Y), \quad Y = f(x, A),$$

where $\mathcal{F}(Y)$ is the set of all fuzzy subsets on Y . The fuzzy set Y is defined by the membership function

$$\mu_Y(y) = \begin{cases} \max \mu_A(a) & \text{for } \{a \mid y = f(x, a)\} \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases}$$

The estimation problem is basically a problem of finding estimates for the parameters A_p .

one takes estimate A_p which minimizes an L_1 norm.

$$J(A) = \|y - \hat{y}\|_1 = \sum_{k=1}^M |y_k - \hat{y}_k| \quad (3)$$

Then the fuzzy linear regression model can be rewritten as follows:

$$\hat{y} = Y^* = (\alpha_0, c_0)\varphi_0(x) + (\alpha_1, c_1)\varphi_1(x) + (\alpha_2, c_2)\varphi_2(x) + \dots + (\alpha_p, c_p)\varphi_p(x), p = 1, 2, \dots, n_a + n_b \quad (4)$$

The following linear programming (LP) formulation was employed to estimate Fuzzy parameters as follows

$$A_p = (\alpha_p, c_p)$$

$$\text{minimize } J = \sum_{j=0}^p \left(c_j \sum_{i=1}^m \varphi_j(x_i) \right) \quad (5)$$

$$\text{Subject to, } \sum_{j=0}^p \alpha_j \varphi_j(x_i) + (1-h) \sum_{j=0}^p c_j \varphi_j(x_i) \geq y_i$$

$$\sum_{j=0}^p \alpha_j \varphi_j(x_i) - (1-h) \sum_{j=0}^p c_j \varphi_j(x_i) \leq y_i \quad \alpha_j \in R, c_j \geq 0 \quad j = 0, 1, 2, \dots, p$$

where J is the total fuzziness of the fuzzy regression model. The h -value is between 0 and 1, which is h threshold level to be chosen by the decision maker. This term is referred to as a degree of fitness of the fuzzy linear model to its data.

If this approach is applied to the controlled plant with ARX model, estimated controlled values of the system are obtained as follows.

$$\hat{y}_i = \sum_{j=0}^p \alpha_j^T \varphi_j(x_i) \quad (6)$$

Thus, for a sequence of last M measurements from the ith instant, the output estimate for the regression model can be written as:

$$y_i = \hat{y}_i + e_M \quad (7)$$

$$y_i = F_M \alpha^T + F_M c^T$$

$$F_M = \begin{bmatrix} \varphi^T(x_1) \\ \varphi^T(x_2) \\ \vdots \\ \varphi^T(x_M) \end{bmatrix} = \begin{bmatrix} \varphi_1(x_1) & \varphi_2(x_1) & \cdots & \varphi_k(x_1) \\ \varphi_1(x_2) & \varphi_2(x_2) & \cdots & \varphi_k(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ \varphi_1(x_M) & \varphi_2(x_M) & \cdots & \varphi_k(x_M) \end{bmatrix}$$

$$y_M = \begin{bmatrix} y(k) \\ \vdots \\ y(k-M) \end{bmatrix} \quad e_M = \begin{bmatrix} e(k) \\ \vdots \\ e(k-M) \end{bmatrix}$$

$$= \begin{bmatrix} y(k-1) & \cdots & u(k-1) & \cdots \\ y(k-2) & \cdots & u(k-2) & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ y(k-M+1) & \cdots & u(k-M+1) & \cdots \end{bmatrix} A_p = \begin{bmatrix} (\alpha_1, c_1) \\ \vdots \\ (\alpha_p, c_p) \end{bmatrix} = [a_{(\alpha_1, c_1)} \cdots a_{(\alpha_{na}, c_{na})} \cdots b_{(\alpha_1, c_1)} \cdots b_{(\alpha_{nb}, c_{nb})}]^T$$

$$p = na + nb$$

J is minimized subject to:

$$1 - \frac{|y - \alpha^T \varphi|}{c^T |\varphi|} \geq h \quad (8)$$

$$\mu_{Y^*}(y) = \begin{cases} 1 - \frac{|y - \alpha^T \varphi|}{c^T |\varphi|}, & \varphi \neq 0, \\ 1, & \varphi \neq 0, y = 0, \\ 0, & \varphi \neq 0, y \neq 0 \end{cases} \quad (9)$$

$|\varphi| = [|\varphi_1|, |\varphi_2|, \dots, |\varphi_k|]$ the central value of Y_k^* is $\alpha^T \varphi$ and spread of Y_k^* is $c^T |\varphi|$

When we consider for the set of observed values of nonfuzzy input and nonfuzzy output of the process with the expression

$Y^* = f(\varphi, A)$ the symmetrical triangular membership functions before as described

In vector notation, the fuzzy parameter A can be written as

With $\alpha = (\alpha_1, \dots, \alpha_p)^T$ $c = (c_1, \dots, c_p)^T$ obviously, when $c_j = 0$

reduces to an ordinary number, like general ARX model parameters. And then this model can be applied any MPC technique to predict the output of the process.

FUZZY L1 norm CONTROLLER DESIGN:

Most single-input single-output (SISO) plants, when considering operation around a particular set point and after linearization can be described by

$$A(z^{-1})y(t) = z^{-d}B(z^{-1})u(t-1) + C(z^{-1})e(t) \quad (10)$$

Where $u(t)$ and $y(t)$ are the control and output sequence of the plant and $e(t)$ is a zero mean white noise. A, B ve C are the following polynomials in the backward shift operator z^{-1} :

Where d is the dead time of the system. This model is known as a Controller Auto-Regressive Moving Average (CARMA) model.

It has been argued[26] that for many industrial applications in which disturbances are non-stationary an integrated CARMA(CARIMA) model is more appropriate. A CARIMA model is given by

$$A(z^{-1})y(t) = B(z^{-1})z^{-d}u(t-1) + C(z^{-1})\frac{e(t)}{\Delta} \quad \Delta = 1 - z^{-1} \quad (11)$$

For simplicity in the following the C polynomial is chosen to be 1. Notice that if C^{-1} can be truncated it can be absorbed into A and B

To establish the Fuzzy L1 norm control algorithm is supposed that a model of the linearized plant is expressed in terms of the following FCARIMA model general form.

$$A(\alpha, c)(z^{-1})\Delta y(t) = B(\alpha, c)(z^{-1})\Delta u(t+1) + C(\alpha, c)(z^{-1})e(t) \quad (12)$$

Definition 1: FCARIMA model in which C^{-1} was truncated can be rewritten as the fuzzy linear regression model and on the basis of this strategy, the problem is reduced to an associate discrete-time Fuzzy L1 norm regulation problem for the performance index, for which a fictitious static-state feedback controller needs to be computed. Proposed equation is shown as follows

$$\hat{y} = Y^* = A_0 \Delta \varphi_0 + A_1 \Delta \varphi_1 + \dots + A_{na} \Delta \varphi_{na} + \dots + A_p \Delta \varphi_p \quad (13)$$

$$\text{minimize } J = \sum_{j=0}^P \left(c_j \sum_{i=1}^M \Delta \varphi_j(x_i) \right), \text{ the sum of all deviations is minimal}$$

Where J is the total fuzziness of the FCARIMA model. If we write again J clearly,

$$\text{minimize } J = \sum_{jy=1}^{na} \left(c_{jy} \sum_{k=1}^M (y(k - jy) - w(k)) \right) + \sum_{ju=1}^{nb} \left(c_{ju} \sum_{k=1}^M \Delta u(k - \tau_p - ju + 1) \right) \quad \text{Where } w(k) \text{ is the reference trajectory point}$$

J is minimized subject to,

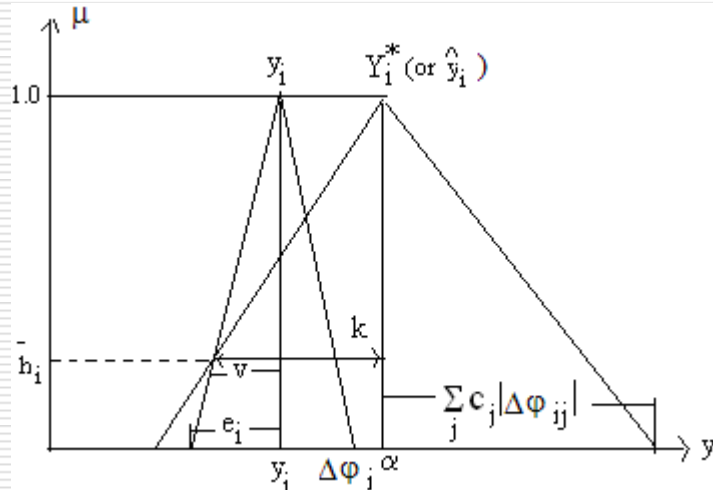
$$\sum_{jy=1}^{na} \alpha_{jy} (y(i - jy) - w(i)) + \sum_{ju=1}^{nb} \alpha_{ju} \Delta u(i - \tau_d - ju + 1) + (1 - h) \left[\sum_{jy=1}^{na} c_{jy} (y(i - jy) - w(i)) + \sum_{ju=1}^{nb} c_{ju} \Delta u(i - \tau_d - ju + 1) \right] \geq y_i \quad (14)$$

$$\sum_{jy=1}^{na} \alpha_{jy} (y(i - jy) - w(i)) + \sum_{ju=1}^{nb} \alpha_{ju} \Delta u(i - \tau_d - ju + 1) - (1 - h) \left[\sum_{jy=1}^{na} c_{jy} (y(i - jy) - w(i)) + \sum_{ju=1}^{nb} c_{ju} \Delta u(i - \tau_d - ju + 1) \right] \leq y_i, i = 1, \dots, M$$

Where M is the number of samples and h is the degree of fitting chosen by the decision-maker. The degree of fitting, h for a fuzzy linear system as opposed to R^2 for non-fuzzy linear regression which ranges between 0 and 1 is an index which indicates that the observation y_i is contained by fuzzy estimation with more than h degrees,[25]

$$1 - \frac{|y - \alpha^T \Delta \varphi(x_i)|}{c^T |\Delta \varphi(x_i)|} \geq h \quad \mu_{y^*}(y) = \begin{cases} 1 - \frac{|y - \alpha^T \Delta \varphi|}{c^T |\Delta \varphi|}, & \Delta \varphi \neq 0, \\ 1, & \Delta \varphi \neq 0, y = 0, \\ 0, & \Delta \varphi \neq 0, y \neq 0 \end{cases} \quad (15)$$

A typical representation of is illustrated in Fig 2.



$$\frac{1 - \bar{h}_i}{1} = \frac{k}{\sum_j c_j |\Delta \varphi_{ij}|},$$

$$k = |y_i - \alpha^T \Delta \varphi_i| + e_i (1 - \bar{h}_i)$$

$$\bar{h}_i = 1 - \frac{|y_i - Y_i^*|}{\sum_j c_j \Delta \varphi_{ij} - e_i}$$

$$\frac{v}{e_i} = \frac{1 - \bar{h}_i}{1} \quad v = e_i (1 - \bar{h}_i)$$

Fig: 2. h_i is derived using the similarity of the right triangles.

Where, \bar{h}_i is the degree of fit of the fuzzy linear regression model to the sample i , and the degree of the fit for the whole model is the minimum of the h_i 's for all samples. In this correspondence, "The primal problem" of solution of standard form of the simplex method is selected and used an on-line adaptation for it. The L1 technique using the simplex method of solution could as well solve linear approximation problems with additional linear constraints

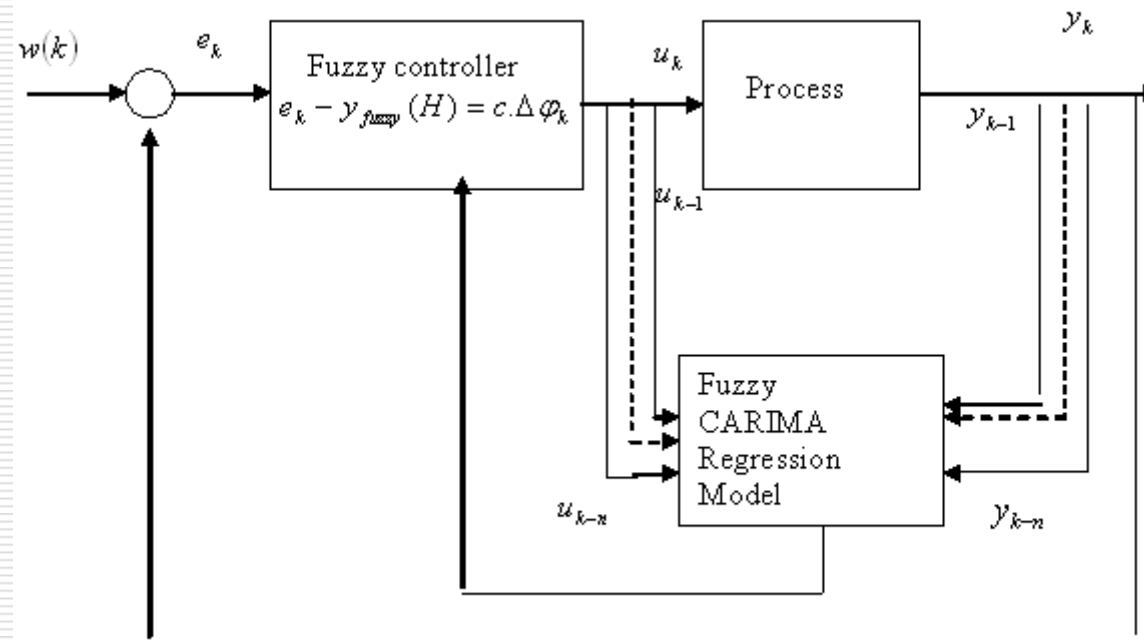


Fig. 3: Fuzzy L_1 norm controller block diagram for general model based control strategy form.

According to blok diagramme the problem considered in this section is essentially the problem of finding an optimal $u_{opt}(kT_0) \in \mathfrak{R} \quad k \geq 0$ fuzzy L1 optimal regulation problem considered here is reduced to the determination of the optimal gains of process . The solution is given by

$$y_k - w(k) - y_{fuzzy}(h) = c_{a_1} e_{k-1} + \dots + c_{a_{na}} e_{k-na} + c_{b_1^*} \Delta U_{k-\tau_d} + c_{b_2} \Delta U_{k-1-\tau_d} + \dots - c_{b_{nb}} \Delta U_{k-nb-\tau_d} \quad (16)$$

This equation upon rearrangement becomes:

$$\Delta U_k = (e_k - c_{a_1} e_{k-1} - \dots - c_{a_{na}} e_{k-na} - c_{b_2} \Delta U_{k-\tau_d-1} - \dots - c_{b_{nb}} \Delta U_{k-nb-\tau_d}) / c_{b_1} \quad (17)$$

Control motion is said to be asymptotically generated in the system , if the state variables

$$\text{fullfills} \quad \lim_{t \rightarrow \infty} c^T \Delta \varphi_k = 0 \quad (18)$$

Fuzzy L_1 norm motion is generated either in finite time ,
If $e(y,t) = 0$ or asymptotically generated; i.e., $e(y,t) \rightarrow 0$ as $t \rightarrow \infty$. ■

Here h represents the minimum degree of certainty acceptable, and we will refer to this interval as h - certain observed interval.

Given a symmetric triangular fuzzy number for y_i , if we are only interested in that part of y_i which has a membership value of at least h , $0 \leq h \leq 1$, we should use the interval segment in Fig.4

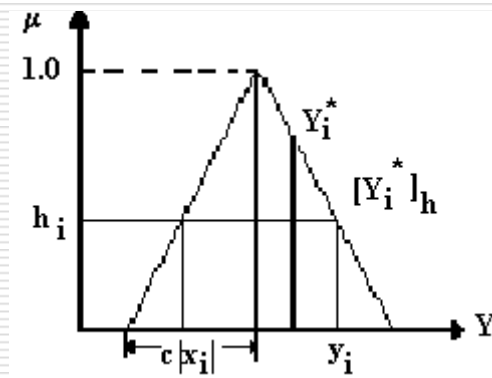


Fig.4.Memberships of y_i

The approach of Peters [34]

In order to treat manipulated variable as smoothly, Peters modified Tanaka's approach by treating the bounds of the interval as fuzzy. In his model, he introduced a new variable which represents the membership degree to which the solution belongs to the set "good solution". The fuzzy linear programming problem is presented for model based control as follows:

Maximize λ

Subject to

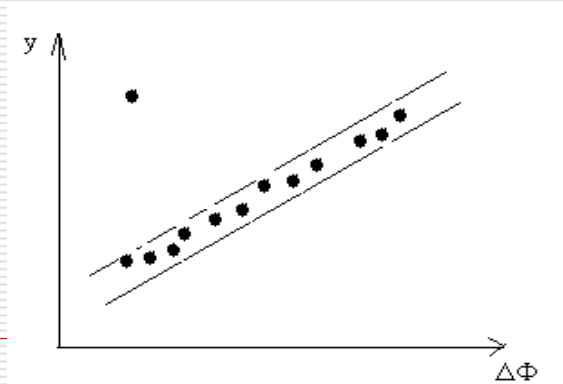
$$(1-\lambda)p_0 - \sum_{i=1}^M c^t |\Delta\varphi_i| \geq -d_0 \quad (\text{"objective function"})$$
$$(1-\lambda)p_i + \alpha^t \Delta\varphi_i + (1-H)c^t |\Delta\varphi_i| \geq y_i \quad (\text{"upper limit"})$$
$$(1-\lambda)p_i - \alpha^t \Delta\varphi_i + (1-H)c^t |\Delta\varphi_i| \geq -y_i \quad (\text{"lower limit"})$$
$$-\lambda \geq -1, \lambda, c \geq 0, i = 1, 2, \dots, M$$

Where d_0 represents the desired value of the objective function, p_0 can be considered as the tolerance of the desired lower bound and p_i as the width of the tolerance of y_i .

$$e_i \bar{e}_i - \sum_{i=1}^M c^t |\Delta \varphi_i| \geq -d_0$$

To width of the estimated interval depends on the selection of parameters d_0, p_0, p_i and strong requirements to minimize the spread (a low value of p_0 and high values of p_i) lead to a narrow interval

, Peters suggested to select $d_0=0$ which makes $\sum_{i=1}^N c^t |\Delta \varphi_i| = 0$ for desired value of the total vagueness of the $(y_i - w(t))$.



ILLUSTRATIVE EXAMPLES

Example 1:

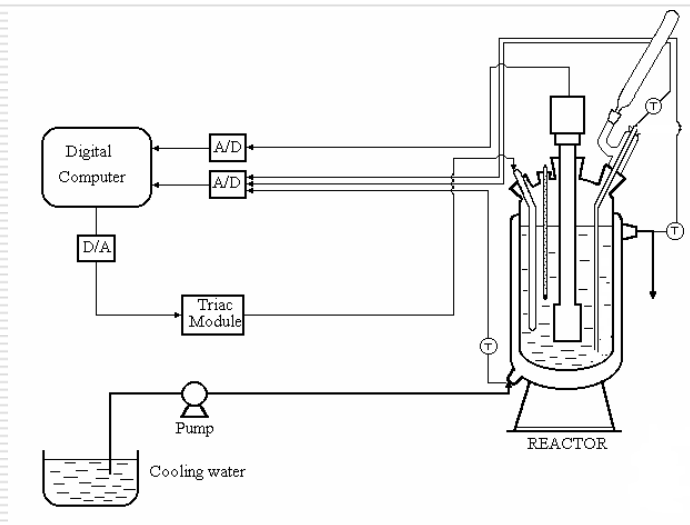


Fig.5 Experimental set-up

At the start of the experiment, the reactor content was heated to the desired temperature. Then, Cooling water was pumped to the jacket at a certain temperature as a disturbance effect. Next, the control methods were applied in the control system of the reactor in order to keep the reactor temperature at the desired value. Heat input from immersed heater was adjusted by Triack Module and it was chosen as a manipulated variable.

We make the following assumptions:

- . The volume and liquids are constant with constant density and heat capacity.
- . Perfect mixing is assumed in both the tank and jacket
- .The tank inlet and outlet flow rate are constant, jacket inlet temperature may change.

Recall that our two dynamic functional equations are:

Energy balance for reactor

$$V\rho C_p \frac{dT}{dt} = Q - UA(T - \bar{T}_c) \quad \bar{T}_c = \frac{T_{ci} + T_{c0}}{2}$$

energy balance for cooling jacket:

$$V_c \rho_c C_{p_c} \frac{dT_{c0}}{dt} = \dot{m}_c C_{p_c} (T_{ci} - T_{c0}) + UA(T - \bar{T}_c)$$

Parameters and Steady-State Values:

$$Q = 9028.62 \text{ Cal/s}^\circ\text{C} \quad UA = 9091.8 \text{ Cal/s}^\circ\text{C}$$

$$T_{ci} = 24.0^\circ\text{C} \quad T = 85.0^\circ\text{C}$$

$$T_{c0} = 75.0^\circ\text{C} \quad \dot{m}_c C_{p_c} = 6300 \text{ Cal/s}$$

Example 1 were identified. The test signal was a PRBS with maximum length 51, amplitude 2000 watt and mean value 1000. The minimum switching time of the test signal was $51 \times 1 = 51$ s. $N = 51$ data, were used for the identification. The dynamic response of reactor temperature according to heat inputs, PRBS signals are seen in Fig 6-9. Before starting the Fuzzy I1 norm identification, all possible model components regressed individually with the output signal that means 4 different submodels, each with five unknown parameter, were identified. Based upon the computed standard deviations of the residuals, the model components could be arranged in a queue, starting with the most important components. The possible model components were chosen as eqn.16-19. Table 1,2,3, and 4 show the model components parameters in the sequence as they were involved in the model.

Model 1:

$$y_k = A_0 + A_1(y_{k-1} - w) + A_2(y_{k-2} - w) + A_3 \sqrt{(u_{k-1} - u_{ss})} \quad (16)$$

Model 2:

$$y_k = A_0 + A_1(y_{k-1} - w) + A_2(y_{k-2} - w) + A_3(y_{k-3} - w) + A_4(u_k - u_{ss}) + A_5(u_{k-1} - u_{ss}) \quad (17)$$

Model 3:

$$y_k = A_1 w_k + A_2 y_{k-2} + A_3 u_{k-1} \quad (18)$$

Model 4:

$$y_k = A_0 + A_1(y_{k-1} - w) + A_2(y_{k-2} - w) + A_3(y_{k-3} - w) + A_4(u_k - u_{ss}) + A_5(u_{k-1} - u_{ss}) \quad (19)$$

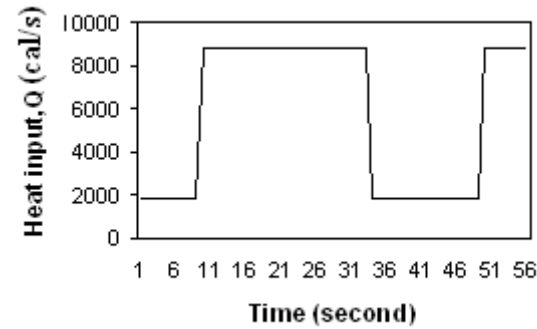
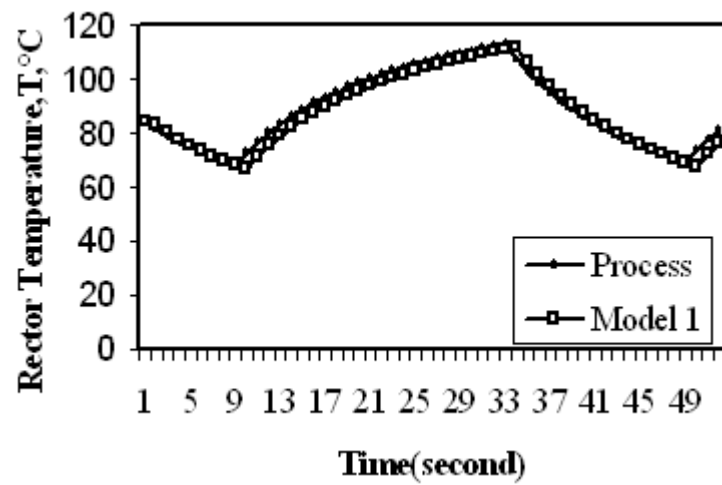


Fig.6. Measured output and computed signals of process 1 with input for the estimated model 1

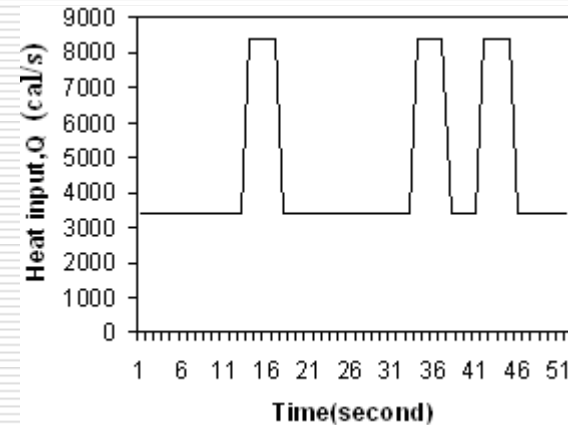
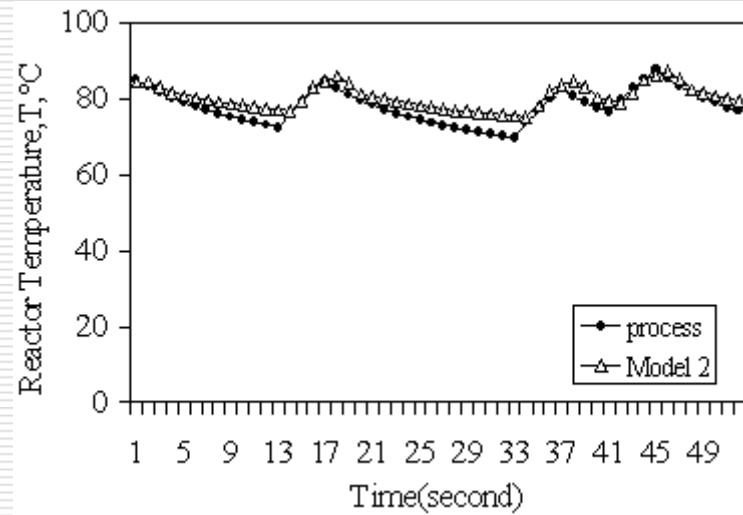


Fig.7 Measured output signals and computed signals of process 1 with input for the estimated model 2

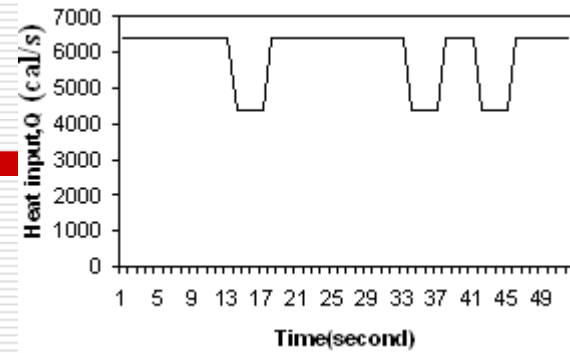
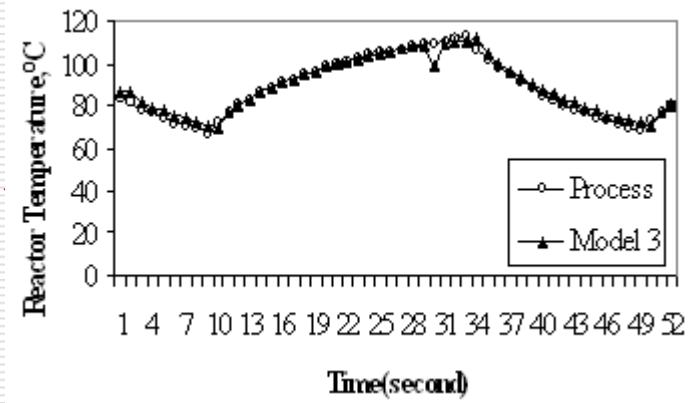


Fig. 8 Measured output signals and computed signals of process 1 with input for the estimated model 3.

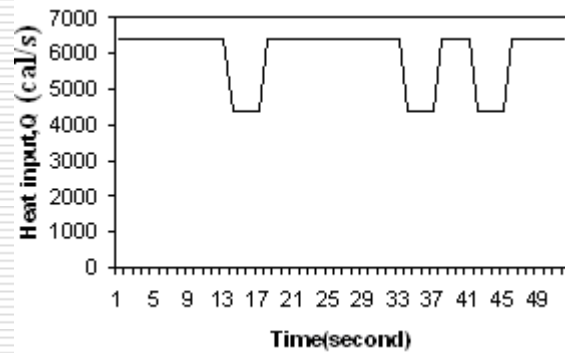
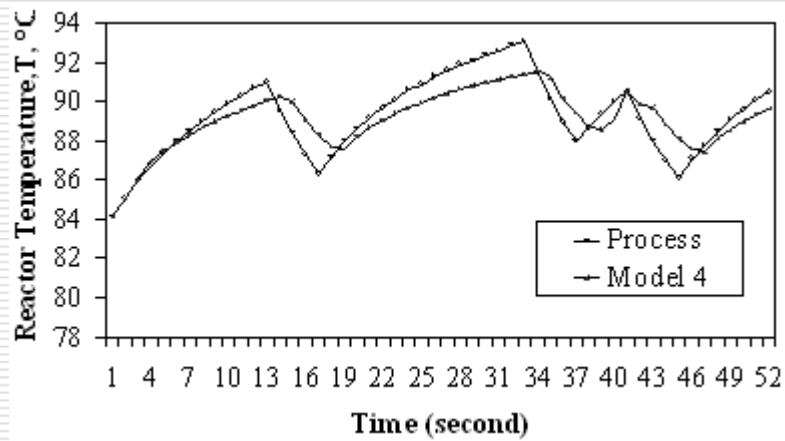


Fig.9. Measured output signals and computed signals of process 1 with input for the estimated model 4.

Table 1,2,3, and 4 show the model components parameters in the sequence as they were involved in the model.

Table 1. Fuzzy Parameters of Well Mixed Reactor with Cooling Jacket $A^*(H=0.5)$ for Model1

Fuzzy Parameters	A_0^*	A_1^*	A_2^*	A_3^*	A_4^*	A_5^*
center, α_i	83.75541	1.00593	0.00	0.00	-	-
width, c_i	76.37967	0.0	0.00	0.48418	-	-

Table 2. Fuzzy Parameters of Well Mixed Reactor with Cooling Jacket $A^*(H=0.5)$ for Model 2

Fuzzy Parameters	A_0^*	A_1^*	A_2^*	A_3^*	A_4^*	A_5^*
center, α_i	84.48534	0.52382	0.00	0.00	0.00017	0.00040
width, c_i	77.53319	0.0	0.00	0.0	0.00041	0.00085

Table 3. Fuzzy Parameters of Well Mixed Reactor with Cooling Jacket $A^*(H=0.5)$ for Model 3

Fuzzy Parameters	A_0	A_1	A_2	A_3	A_4	A_5
center, α_i	-	0.21914	0.79707	0.00109	-	-
width, c_i	-	0.16996	0.81225	0.0	-	-

Table 4. Fuzzy Parameters of Well Mixed Reactor with Cooling Jacket $A^*(H=0.5)$ for Model 4

Fuzzy Parameters	A_0	A_1	A_2	A_3	A_4	A_5
center, α_i	86.63158	0.01189	0.57916	0.0	0.00022	0.00006
width, c_i	78.18326	0.01070	1.1995	0.0	0.00029	0.000

The input-output data shown in Table 3 were formatted model 3 for fuzzy linear regression model. After the transition of the membership functions, the 51 data sets were substituted into the linear programming model which was solved using SIMPLEX algorithm. Table 4 indicates the centers and widths of each fuzzy parameter where $H=0.5$ indicates that the regression model can the data set greater than 50%, i.e., half of the variation will be included by the model. The center and width of the constant (A_3) are both zero which represents the condition that these regression models pass the primary output or are not decided by the centers and width of the parameters A_3 . In table 2. A_2 and A_3 are zero. it means total construction of output are not only proportional to y_{k-2} and y_{k-3} , but other parameters also have small widths of parameters which indicated lower levels of fuzziness.

Control Results For Example I

The model 2 was chosen as the best model for fuzzy I1 norm temperature control of the batch reactor .

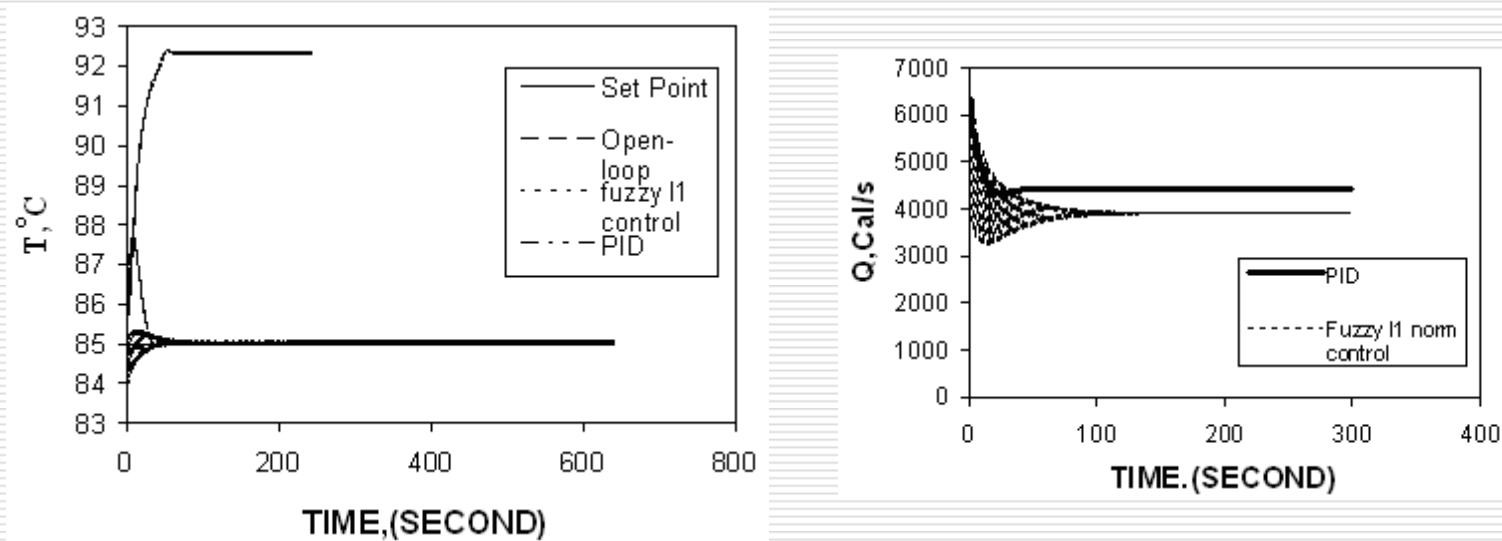


Fig 10: a) Reactor temperature b) Heat input profiles with fuzzy I1 norm and PID.

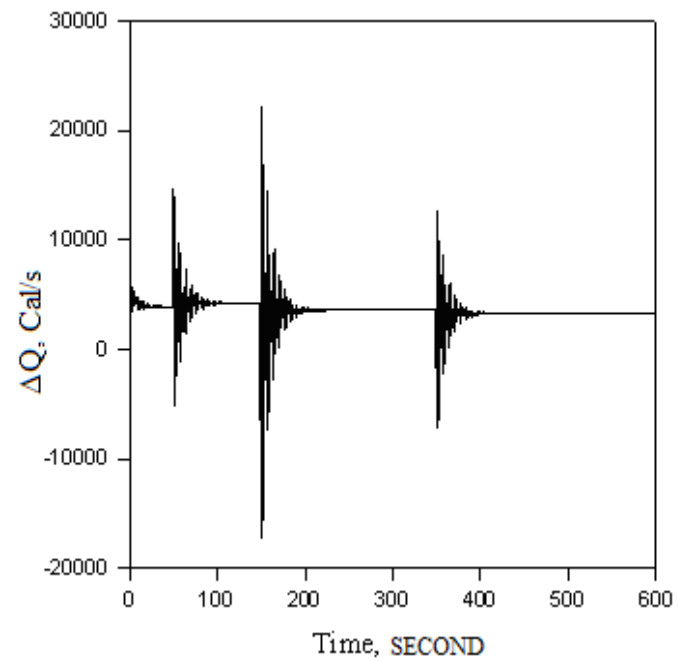
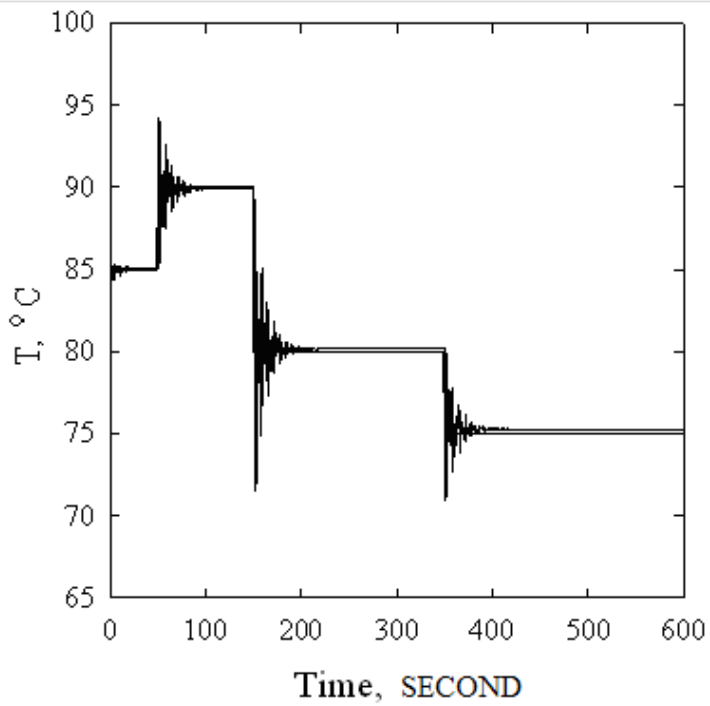


Fig.11. Reactor temperature tracking and ΔQ heat input profiles with fuzzy $l1$ norm control

Example II

Risk is the probability of facing dangerous consequences and situations. Although the chemical processes are safe to operate, people find them dangerous. Because the accidents that happen in a chemical factory threaten the environment and risks are involved in chemical processes.

Therefore, this paper presents a fuzzy linear programming control methodology for the automatic generation of accident scenarios in a sulfur plant. The control strategy suggested here relies on solving the continuous L1 regulation problem. The simulation of the accident scenarios is a time and labor consuming procedure and useful in the detection and the identification of dangerous states in such complex chemical plants during the design stage. Also The purpose of this study is to develop an efficient fuzzy control which can prevent blockage in the valves in case that elemental sulfur freeze.'

Hazard and Operability Method (HAZOP) study performed during the assembly stage is shown in the gases that may develop, are removed by the absorbing gases flow to the tank (N₂). As there is an open valve down into the dissolved sulfur, there is no possibility of increase in the pressure. But the high pressure may be seen along the down liquid feeding pipe (5 m.) and if the sulfur becomes rigid, it may block the valve. The steam shall be switched off or the heaters shall be turned off for the rigidity. The HAZOP study may be performed in accordance with the viscosity of the sulfur is shown in Table 5. This table helps to select the manipulated variable and control variable of sulfur tank problem.

Table 5: The risk analysis of the sulphur plant.by Şahin M. (1998)

Guidelines Word	Parameter	Cause	Effect	Measure
More	Pressure	1.Fire 2.Vapour/N ₂ line may be open 3.Vapour jacket may be suffocated or defect 4.The outlet valve may be closed.	1.Sulphur leakage 2.The manufacturing has to be stopped	1.Reducing of the pressure

In this work, we address the controllable and observable, unstable third-order continuous-time system with transfer function attributing to liquid level of such a sulfur tank[32].

$$y(s) = \frac{-0.5s + 1}{s(s^2 + 1)} e^{-s} u(s) \quad (20)$$

Where

$y(s)$ =Liquid level of sulfur tank at Laplace domain
 $u(s)$ = manipulated variable of the system as a pressure at Laplace domain.

Operating conditions of Sulfur tank are:

Tmax: 204 °C Pmax: 2.691 kPa. μ_{\max} :90000cp
Tmin: 149-177 °C , Pmin: 1.42x10⁻³ kPa μ_{\min} : 4 cp

Control Results For Example 2

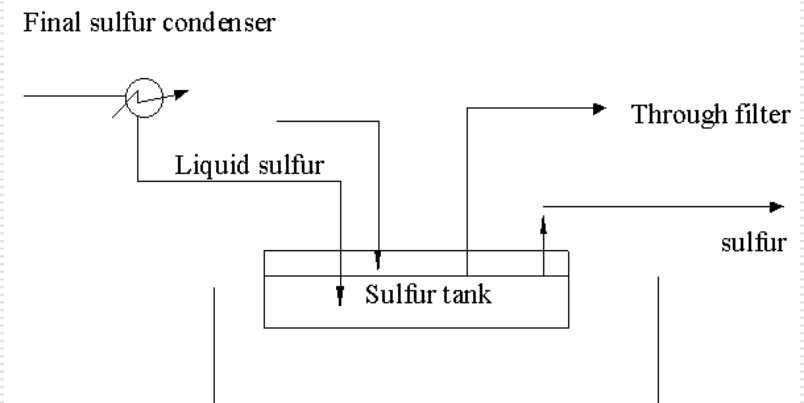
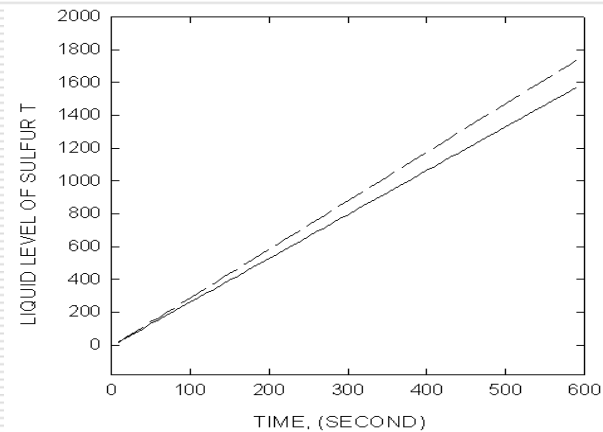


Fig. 12 Performance of the estimated model of system with fuzzy $I1$ norm, Liquid level(m.)

$$Y_k^* = A_0^* + A_1^* e_{k-1} + A_2^* e_{k-2} + A_3^* e_{k-3} + A_4^* \Delta U_k + A_5^* \Delta U_{k-1} \quad (21)$$

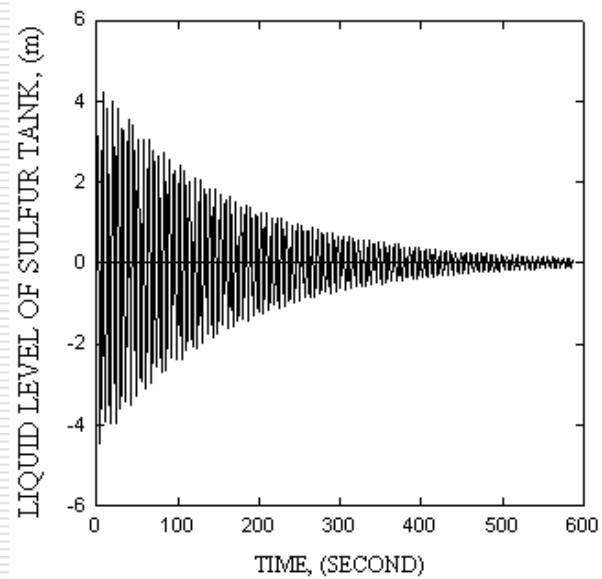
where y_k : Liquid level of sulfur tank and pipe (m.)

ΔU_k : The Change of controller output as the system pressure (kPa).

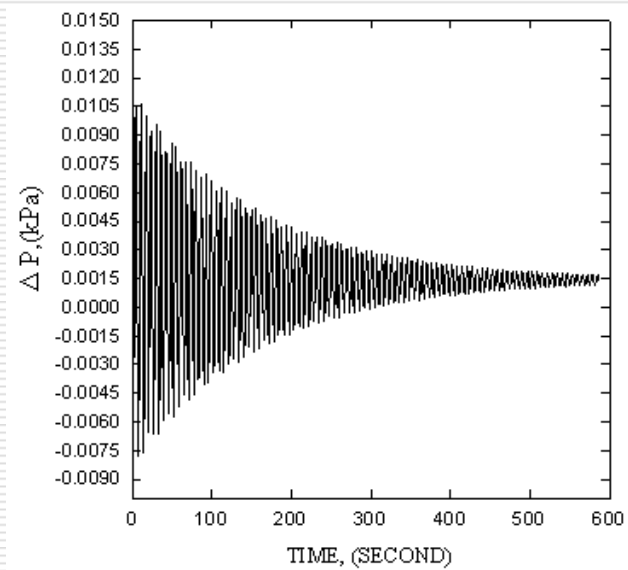
e_k : $y_k - y_{set}$

Table 6. Fuzzy Parameters A^* (H=0.5) for Eq. 21

Fuzzy Parameters	A_0^*	A_1^*	A_2^*	A_3^*	A_4^*	A_5^*
center, α_i	0.32619	1.10474	0.00	0.00303	0.0	0.0002076
width, c_i	11.45476	3.2599	0.00	1.4509	0.0012892	0.00047621



a)



b)

Fig .13. (a)Output:tracking of liquid level for sulfur tank , (b). Input:The changes of system pressure for Fuzzy I_1 norm control

When we apply The Modification of outlier classifier to calculate the manipulated variable, the error function is as follows:

When the value is not too abnormal, it is frequently difficult to decide whether it is an outlier. To solve this problem, we adopt the fuzzy concept and modify the classifier as follows:

$$\alpha^t \Delta \varphi_i + c^t |\Delta \varphi_i| \geq y_i + (1 - \lambda) e_i \quad \alpha^t \Delta \varphi_i - c^t |\Delta \varphi_i| \leq y_i - (1 - \lambda) e_i \quad (22)$$

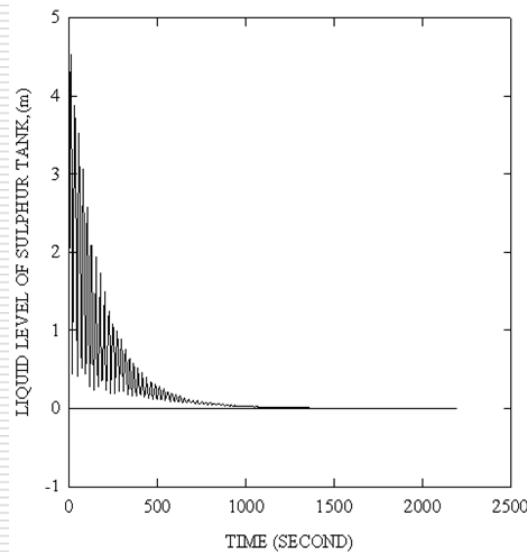
The above equation using $\lambda - cut$ to decide abnormal values. When the membership function of the interested value is larger than λ , this interested value is treated as abnormal value. Generally speaking, abnormal value e_i should be either $e_{i-1} \leq e_i \leq e_{i+1}$ thus we consider as follows:

$$e'_i = (1 - \lambda) e_i = \bar{e} e_i$$

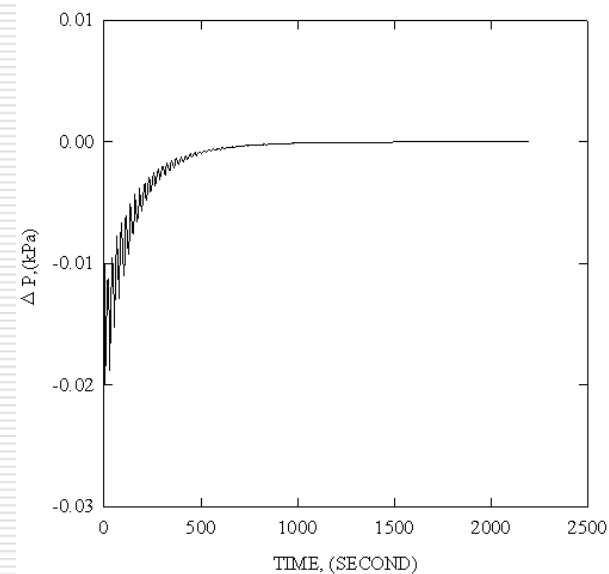
$$c^t |\Delta \varphi_i| \leq e'_i \quad k = \bar{e} = \sum_{i=1}^N e_i / N \quad (23)$$

Obviously, the value of λ decides the influences of the abnormal values on the overall data, or decides the degree of importance of these outliers. If the decision maker thinks the handling of abnormal values are very important, then we must use this approach to control

When we use $c^t \Delta \varphi_i = e_i \bar{e}$ approach for example 2, results are as follows:



a)

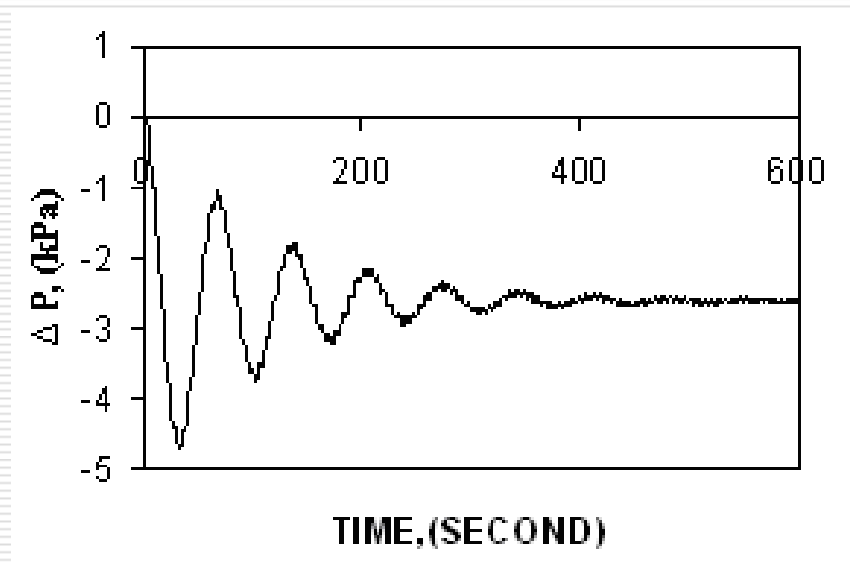
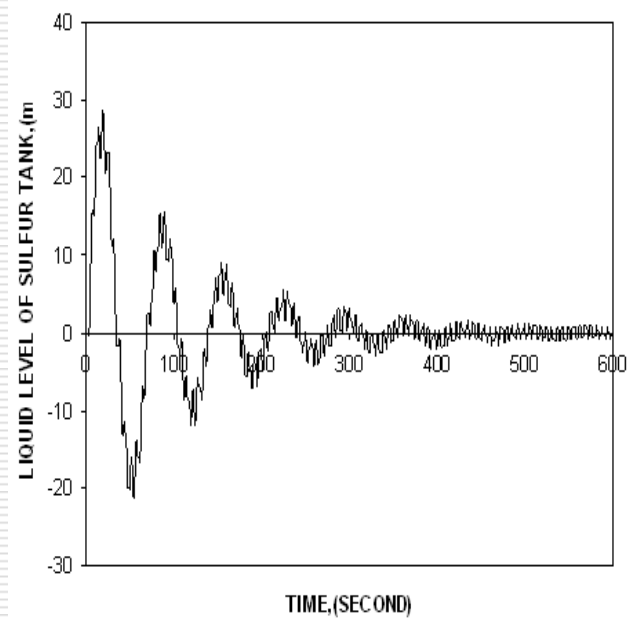


b)

Fig 14: (a) Output: tracking of liquid level for sulfur tank , (b). Input: The changes of system pressure for Modified Fuzzy I_1 norm control

PID controller results:

$$K_C=3 \times 10^{-2}, \tau_I=8 \times 10^{-3}, \tau_D=1 \times 10^{-6}.$$



a)

b)

Fig 15: (a), Output:tracking of liquid level for sulfur tank, (b).Input:The changes of system pressure for PID

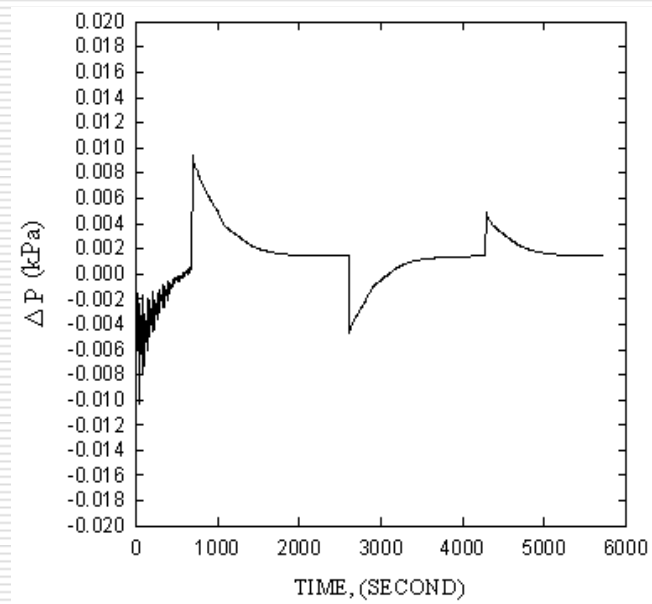
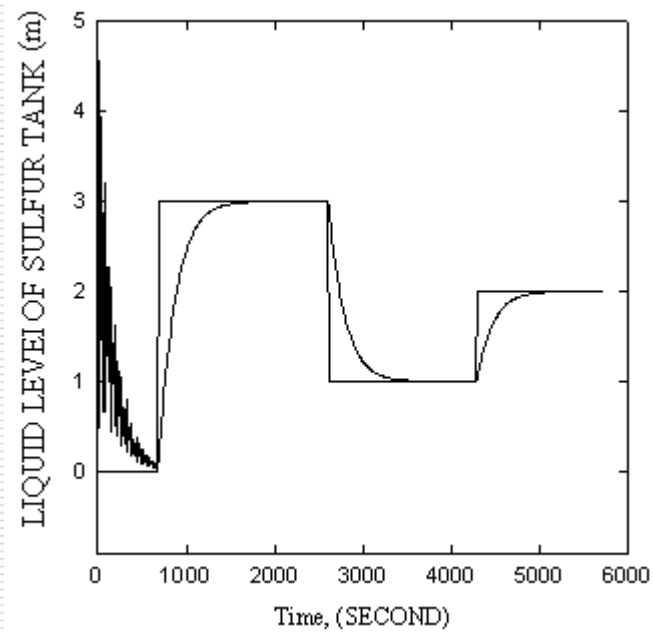


Fig: 16.Set point tracking for the liquid level control of sulfur tank via the impulse disturbance , in the case where $y_{fuzzy}=0$

CONCLUSION

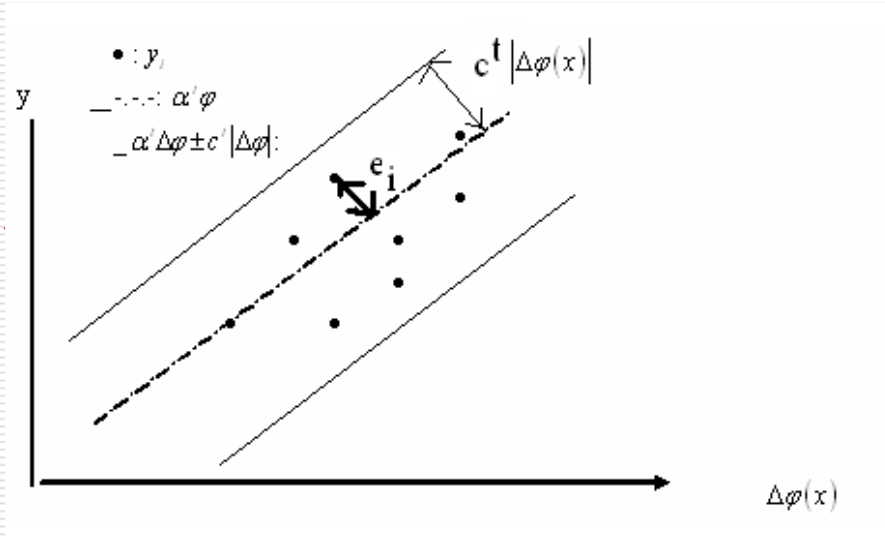


Fig.: Estimated values for . When nonfuzzy process output data is used

$$\text{minimize } J = \sum_{j=0}^P \left(c_j \sum_{i=1}^M \Delta \varphi_j(x_i) \right)$$

$$\sum_{j=0}^P \alpha_j \Delta \varphi_j(x_i) + (1-h) \sum_{j=0}^P c_j \Delta \varphi_j(x_i) \geq y_i$$

$$\sum_{j=0}^P \alpha_j \Delta \varphi_j(x_i) - (1-h) \sum_{j=0}^P c_j \Delta \varphi_j(x_i) \leq y_i$$

When we consider that there is not vagueness or fuzzies, error spreading does not increase with increasing the $\Delta\varphi(x)$

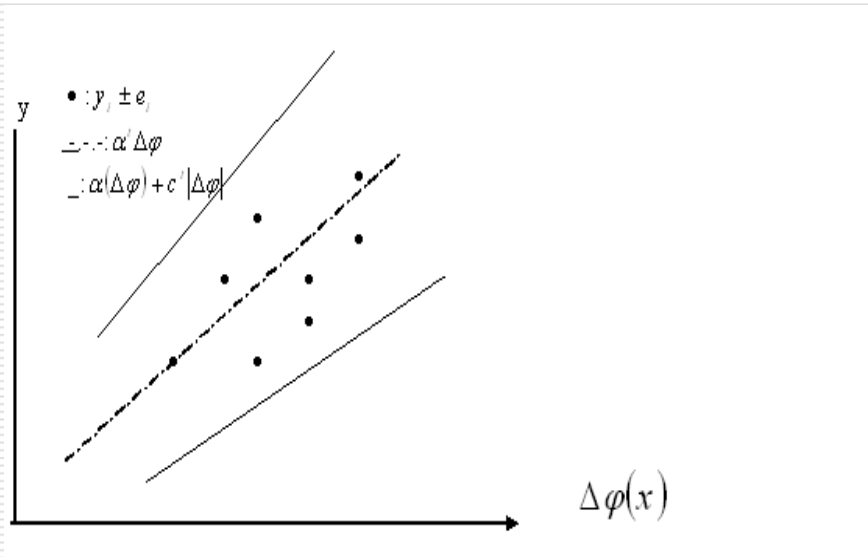


Fig: When fuzzy output data is used, Estimated value for the y case with increasing spread as $\Delta\varphi(x)$ increases.

$$\alpha^t \Delta \varphi_i + (1-h)c^t |\Delta \varphi_i| \geq y_i + (1-h)e_i$$

$$\alpha^t \Delta \varphi_i - (1-h)c^t |\Delta \varphi_i| \leq y_i - (1-h)e_i$$

When the process far from operational condition, vagueness and error spreading, increase with increasing the $\Delta\varphi(x)$

References

- [1] Jeong, J. Song and Sunwan Park, Neural model predictive control for nonlinear chemical process, *Journal of Chemical Engineering of Japan*, 26,4(1993),347-354,.
- [2] Ali, E. and Zafiriou, E., Optimization-based tuning of nonlinear model predictive control with state estimation, *J. Process Control* 3(1993), 97-107.
- [3] Hernandez, E. Arkun, Y., On the global solution to nonlinear model predictive control algorithms that use polynomial models, *Computers & Chem. Eng.* 18(1994), 533-536.
- [4] Bequette, B.W., Nonlinear control of chemical processes: A review, *Ind. Eng. Chem. Res.* 30 (1991), 1391-1453.
- [5] Garcia, C.E., Prett, D.M. and Morari, M., Model predictive control: Theory and practice-a survey, *Automatica* 25 (1989), 335-348.
- [6] Gustafsson, T.K. and Mäkilä, P.M, Modelling of uncertain systems with application to robust process control, *Journal of Process Control*, 11 (2001),3, 251-264.
- [7] Kiparissides, C., Papadopoulos, E. and Morris, J., *Real-Time Optimization and Model-Based Control of Polymer Reactors, Methods of Model Based Process Control*, R. Berber (ed.) Kluwer Academic Publisher, Netherlands]. (1995) 495-530.
- [8] Megias, D., Serrano J. and Prada, C. de, Min-max constrained quasi-infinite horizon "model predictive control" using linear programming, *Journal of Process Control*, vol 12(4)(2002), Pp 495-505
- [9] Peng Ching-Yu and Jang, Si-Shang, Nonlinear Rule Based Model Predictive Control of Chemical Processes, *Ind. Eng. Chem. Res.*, 33 (1994), 2140-2150.
- [10] Barrodale, L. and Roberts, F.D. K., "An improved algorithm for discrete l1 linear approximation," *SIAM J. Numer. Anal.*, vol. 10(1973), pp. 839-848, Oct..
- [11] Presman, E., Sethi, S.P and Zang, Q., "Optimal feedback production planning in a stochastic N-machine flowshop," *Automatica*, vol. 31(1995), pp.1325-1332,.
- [12] Roberts P.D. and Ben-Israel, A., "An interval programming algorithm for discrete linear l1 approximation problems," *J. Approx. Theory*, vol.2, (1969) pp.323-336.
- [13] Ridley, J.N, Shaw, I.S, and Kruger, J.J., Probabilistic fuzzy model for dynamics systems. *Electronics letters*.24(4)(1997):890-892.
- [14] Christopher, V.R., Rawlings, B.J., Linear Programming and Model Predictive Control. *Journal of Process Control* 10 (2000), 283-289.
- [15] Arvanitis, K.G., A new adaptive optimal LQ regulator for linear systems based on two-point multirate controllers, *Control and Intelligent Systems*, Vol.28(2000), No.3.
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- [16] Collins, E.G.JR. and Song, T. 2001. An On-line Adaptation for Discrete I1 Linear Estimation, IEEE Trans.on Automatic Control 29(1),(1984),67-71.
- [17] Zadeh, L.A, Whalen, L.H. On optimal control and linear programming, IRE Trans Auto. Cont., (1962),45-46.
- [18] Heshmathy, B. and Kandel, A., Fuzzy linear regression and its applications to forecasting in uncertain environment. Fuzzy sets and Systems, 15,(1985) pp. 159-191.
- [19] Sugeno, M., Industrial applications of fuzzy control. Elsevier Science Publishers, Netherlands, 1985.
- [20] Suruthar, A. and Chaudhuri, A.S, On-line identification of ARX Model for Glucose-Insulin Interaction in a Diabetic Patient from Input-Output Data, IE(I) Journal-ID, 85(2004),1-7,.
- [21] Beck, M.B., Model Structure Identification from Experimental Data, Theoretical Systems Ecology, Academic Press, 1979.
- [22] Chen, Yun-Shiow, Outliers Detection and Confidence Interval Modification in Fuzzy Regression, Fuzzy Sets and Systems, 119 (2001), 259-272,.
- [23] Hojati, M., Bector, C.R., Smimou, Kamal, A Simple Method for Computation of Fuzzy Linear Regression, European Journal of Operational Research, Computing, Artificial Intelligence and Computer Technology, 166(2005), 172-184,.
- [24] Xue, Yu, Kim, I.S, Son, J.S, Park, C.E, Kim, H.H, Sung, B.S, Kim, I.J., Kim, II.J., Kang, B.Y., Fuzzy Regression Method for Prediction and Control the Bead Width in the Robotic Arc-Welding Process, Journal of Materials Processing Technology, 164-165 (2005), 1134-1139,.
- [25] Tanaka, H., Uejima, S., and Asai, K., , Linear regression analysis with fuzzy model. IEEE, Trans. Systems Man Cybernet, 10(4),(1982),2933-2938.
- [26] Clarke, D.W, Mohtadi, C. and Tuffs, P.S. Generalized Predictive Control Part I. The Basic Algorithm. Automatica, 23(2): (1987),137-148,
- [27] Camacho, E.F and Bordons C, Model Predictive Control, Springer-Verlag London Limited, 1999.
- [28] F. Blanchini, F. Rinaldi, and W. Ukovich, , Least inventory control of multi- storage systems with nonstochastic unknown input," IEEE Trans. Robot. Automat., vol. 13(1997), pp. 633-645, Oct. 1989.
- [30] Kolodji, B.P. Hazard Resolution in Sulphur Plants from Design Through Start up Process Safety Progress, 12(1993),(127-131).
- [31] Zeybek, Z., Environmental Safety and Loss Prevention in Sulfur Tank with Fuzzy L1 Norm Control , Soft Computing in Industry-Recent Applications, Book, Ed.: R. Roy, M.Köppen, S. Ovaska, T. Furuhashi, Springer, F. Hoffmann. (2001),201-209,
- [32] Park, J.H., Sung, S.W. and Lee, I.B, An Enhanced PID Control Strategy for Unstable Processes, Automatica, vol.34(1998),No.6, pp 751-756.
- [33] Şahin M. (1998) Hazard and Operability Analysis (HAZOP) in Chemical Process Industries: A case Study., Ankara University Master Thesis.
- [34] Peters, G, Fuzzy linear regression with fuzzy intervals, Fuzzy Sets and Systems 63,(1994) 44-45.
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