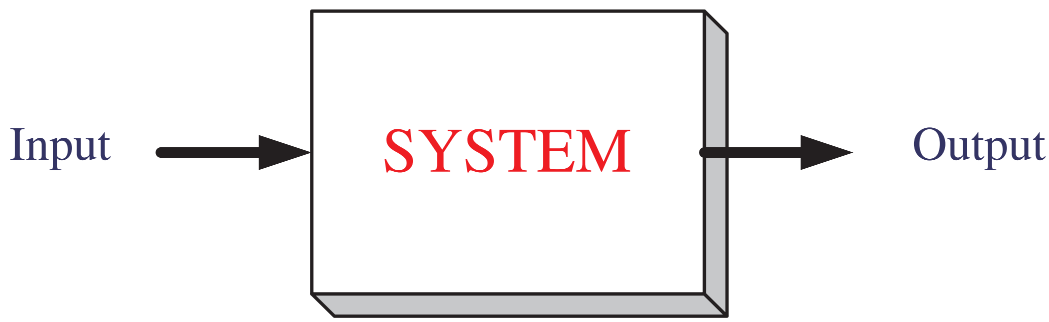
Lecture 0

Motivation and Overview

- What is a System?
- Why Linear Systems?
- Course Plan
- Notation and Background

What is a System?

- Any black box operation that produce output from input.



- **Examples:**
  - A Linear Time Invariant System:

$$\mathbf{y}_n = \sum_{k=-\infty}^{\infty} h_k \mathbf{x}_{n-k}. \tag{1}$$

  - An Analog to Digital Converter: $\mathbf{y}_n = \mathbf{x}(nT)$.
  - Modulator: $\mathbf{y}(t) = \mathbf{x}(t)cos(\omega t)$.
- System Theory deals with the analysis and design of systems based on the analytic and the quantitative models describing the cause and effect relationships and interactions among the system variables.
- The language of system theory is mathematics.

Linear Systems

- What is a Linear System: Two basic properties:
  - Homogeneity: Given a constant c,
  $$\mathcal{F}(c\mathbf{x}) = c\mathcal{F}(\mathbf{x}) \quad (1)$$
  - Additivity:
  $$\mathcal{F}(\mathbf{x}_1 + \mathbf{x}_2) = \mathcal{F}(\mathbf{x}_1) + \mathcal{F}(\mathbf{x}_2) \quad (2)$$

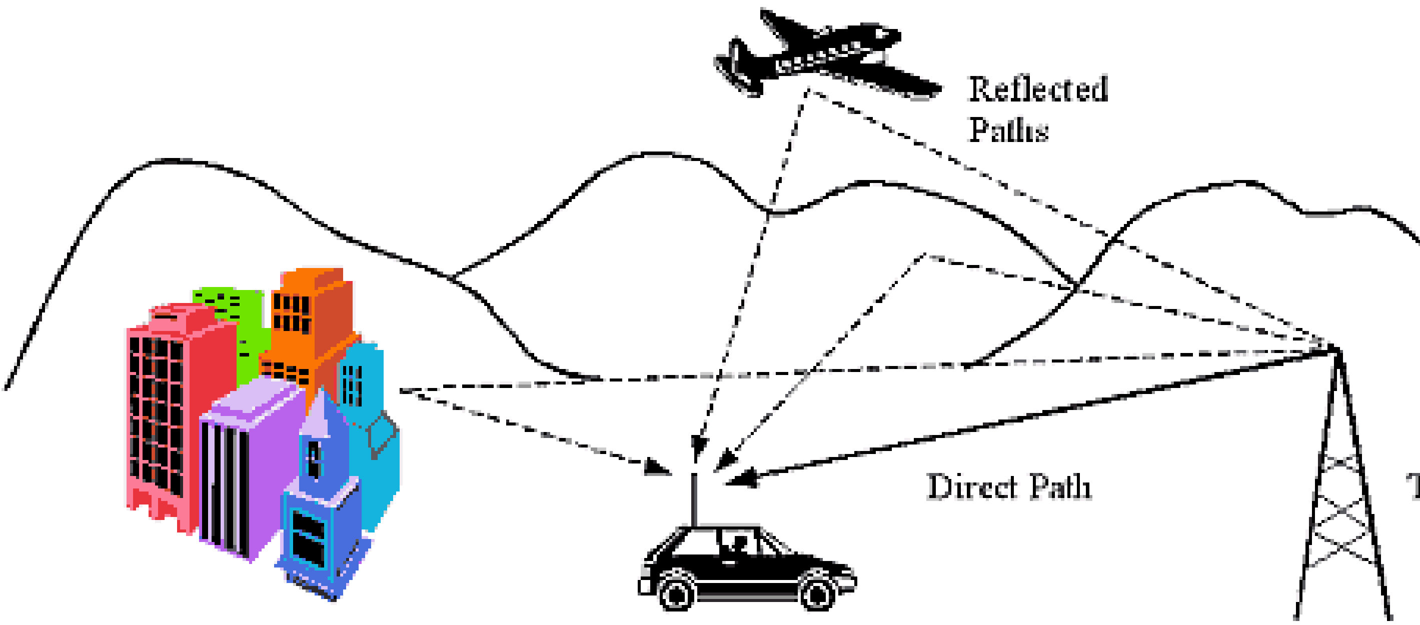
  Combination of these two properties yields the superposition property:
  $$\mathcal{F}(\sum_{i\in\mathcal{I}} c_i\mathbf{x}_i) = \sum_{i\in\mathcal{I}} c_i\mathcal{F}(\mathbf{x}_i). \quad (3)$$

- Why Linear Systems?
  - Mathematically tractable: easier to analyze and implement.
  - Many nonlinear systems can be locally or globally approximated by linear systems.
  - The understanding of the Linear Systems is the prerequisite to the understanding of th Nonlinear Systems.

- Language for Linear System Theory: Linear Algebra (We will spend some time on this).

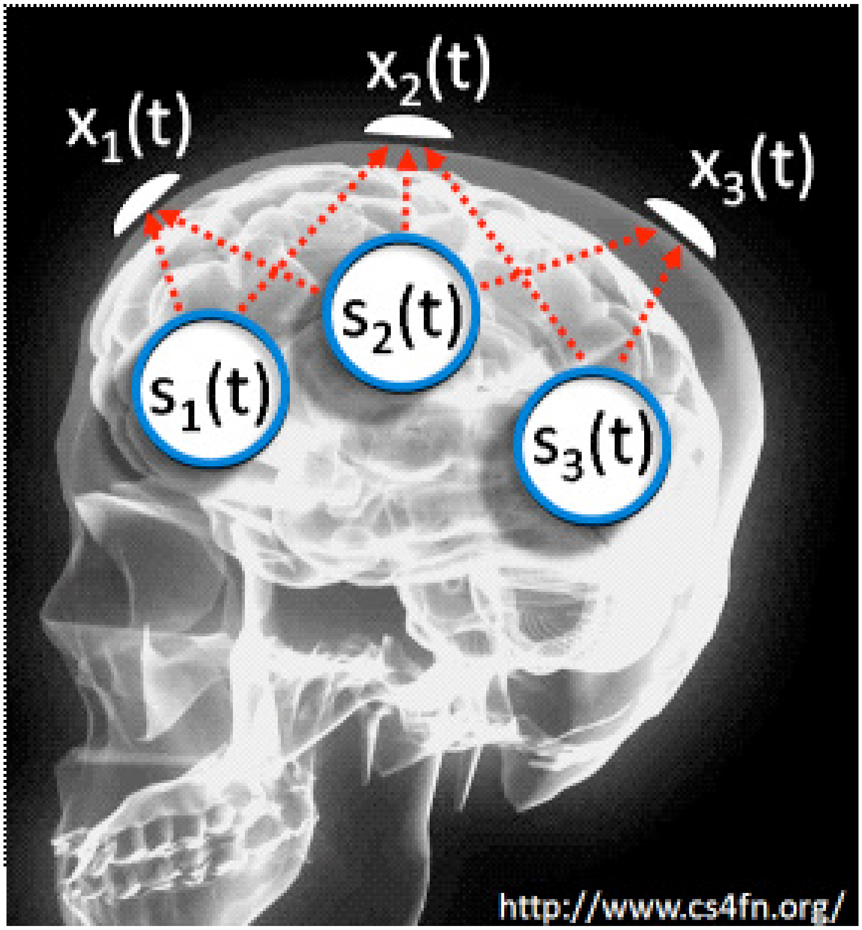
Example

- Wireless Multipath Channel



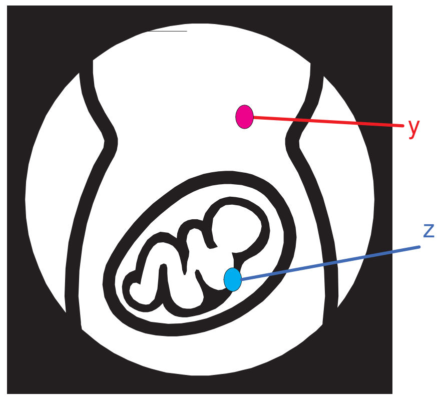
Example

- Brain Activity Monitoring

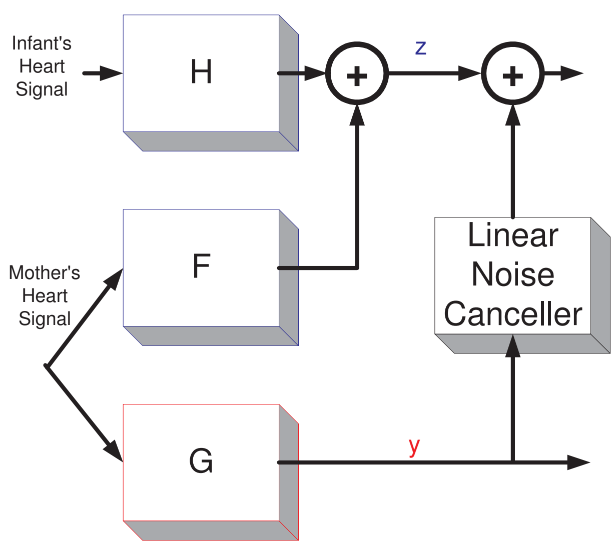


Example

- Monitoring heart-beat of an infant



- The linear model



Course Plan

- Linear Algebra: Review and New Topics
- Inner Product and Norm Spaces
- Singular Value Decomposiition
- Linear Systems and State Space Descriptions
- Time Domain and Frequency Domain Approach
- Observability
- Controllability
- State Space Realizations
- Linear Quadratic Regulator
- Lyapunov Theory

Notation and Background

• Set Descriptions

$\{m_1, m_2, ..., m_n\}$	Specifying list of elements
$\{x \mid P(x)\}$	The collection of elements for which property P holds

• Set Operations

$A \bigcup B$	Union of sets A, B.
$A \bigcap B$	Intersection of sets A, B.
$A \Delta B$	Symmetric Difference, $A \bigcup B - A \bigcap B$.
A^c	Complement of a set $\{x \mid x \notin A\}$.

• Special Sets

$\mathbf{R}$	The set of real numbers.
$\mathbf{R}^n$	the set of real $n \times 1$ vectors.
$\mathbf{R}^{1 \times n}$	the set of real $1 \times n$ vectors.
$\mathbf{R}^{m \times n}$	the set of real $m \times n$ matrices.
$\mathbf{R}_+$	the set of nonnegative real numbers $\{x \mid x \geq 0\}$.
$\mathbf{C}$	The set of complex numbers.
$\mathbf{C}^n$	the set of complex $n \times 1$ vectors.
$\mathbf{C}^{m \times n}$	the set of complex $m \times n$ matrices.
$\mathbf{Z}$	the set of integer numbers.

About Transforms

- Fourier Transform Types:

	Discrete Time	Continuous Time
Finite Duration	$G(f)=\sum_{n=0}^{N-1} g_n e^{-j2\pi fn}$ $f\in\{0,1/N,...,\frac{N-1}{N}\}$	$G(f)=\int_0^T g(t)e^{-j2\pi ft}dt$ $f\in\{\frac{k}{T};k\in\mathbf{Z}\}$
Infinite Duration	$G(f)=\sum_{n=-\infty}^{\infty} g_n e^{-j2\pi fn}$ $f\in[\frac{-1}{2},\frac{1}{2})$	$G(f)=\int_{\infty}^{\infty} g(t)e^{-j2\pi ft}dt$ $f\in\mathbf{R}$

- Unilateral Laplace Transform (defined for causal continous time (CT) sequences):

$$X_-(s)=\mathcal{L}_-\{x(t)\}=\int_{0^-}^{\infty} x(t)e^{-st}dt \qquad (1)$$

Note that the ULT of $\dot{x}(t)$ is

$$\begin{aligned}\int_{0^-}^{\infty}\dot{x}(t)e^{-st}dt &= x(t)e^{-st}|_{0^-}^{\infty}+s\int_{0^-}^{\infty}x(t)e^{-st}dt\\ \mathcal{L}_-\{\dot{x}(t)\} &= sX(s)-x(0)\end{aligned}$$

- Unilateral Z Transform

$$X_-(z)=\mathcal{Z}_-\{x_n\}=\sum_{k=0}^{\infty}x_k z^{-k} \qquad (2)$$

Note that

$$\begin{aligned}\mathcal{Z}_-\{x_{n+1}\} &= \sum_{k=0}^{\infty}x_{k+1}z^{-k}\\ &= z\sum_{k=0}x_k z^{-k}-x(0)\\ &= zX_-(z)-x(0)\end{aligned}$$