ELEC505/ELEC405 Handout #5
Linear Systems Theory Wednesday, November 20, 2013

**Homework Set #4**

Due: Tuesday, December 3, 2013.

Please email your matlab codes to our TA (mkuscu@ku.edu.tr).

1. *Quadratic Forms and Hermitian Matrices*

   Consider the quadratic form

$$f(\mathbf{x}) = \frac{1}{9}(-2x_1^2 + 7x_2^2 + 4x_3^2 + 4x_1x_2 + 16x_1x_3 + 20x_2x_3). \qquad (1)$$

   (a) Find a symmetric matrix $\mathbf{A}$ so that $f(\mathbf{x}) = \mathbf{x}^T\mathbf{A}\mathbf{x}$.

   (b) Diagonalize the quadratic form using the $\mathbf{LDL}^T$ factorization, and determine the inertia of $\mathbf{A}$.

   (c) Is this a positive definite form?

   (d) Verify the inertia obtained above is correct by computing the eigenvalues of $\mathbf{A}$.

2. *Matrix Factorization Coding Experience...*

   (a) Write a Matlab function to find Cholesky Factorization of a positive definite matrix.

   (b) Write another Matlab function to obtain QR factorization based on Gram-Schmidt procedure.

3. *Random Vector Generation...*

   Write a MATLAB function whose first couple of lines are provided as follows

```
function out=generaterandvec(N,R)
% Generates zero-mean complex Gaussian random vector sequence with the specified
% covariance matrix
% Usage
%  out=generaterandvec(N,R)
% Here
% N: Number of vectors to be generated
% R: The specified covariance matrix.
% out: Output matrix containing vector sequence. Assuming R is a pxp matrix
% out would be a pxN matrix
```

   in your function only specialized MATLAB functions you can use are *randn* and *size*. Generate $N = 100000$ random vectors with covariance

$$\mathbf{R} = \begin{bmatrix} 28 & 15+9i & 2+21i \\ 15-9i & 48 & 15-11i \\ 2-21i & 15+11i & 30 \end{bmatrix}. \qquad (2)$$

Calculate the sample covariance matrix for the vector sequence generated using the formula

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N} \mathbf{x}_i \mathbf{x}_i^*. \tag{3}$$

Can you find $\hat{\mathbf{R}}$ without using loops? Compare $\hat{\mathbf{R}}$ with the true covariance $\mathbf{R}$, are they close? does your random generator seem to work fine?

4. Let full QR decomposition of $\mathbf{A} \in \mathcal{C}^{m\times n}$ be given by

$$\mathbf{A} = \mathbf{QR} \tag{4}$$

where $\mathbf{Q} \in \mathcal{C}^{m\times m}$ and $\mathbf{R} \in \mathcal{C}^{m\times n}$. Show that

$$\sum_{k=1}^{m}\sum_{l=1}^{n} |A_{kl}|^2 = \sum_{k=1}^{m}\sum_{l=1}^{n} |R_{kl}|^2 \tag{5}$$