ELEC505/ELEC405 Handout #11
Linear Systems Theory Monday, December 23, 2013

**Homework Set #7**
Due: Optional (before exam), 2013.

1. *Optimal Linear Phase Filter Design*

Suppose we want to design a linear phase filter with impulse response $\{h_{-20}, h_{-19}, ..., h_{-1}, h_0, h_1, h_2, ..$ where $h_i = h_{-i}$ such that we have the following frequency domain pass bands and stop band

Pass Band 1: $0 - 0.3\pi$

Pass Band 2: $0.85\pi - \pi$

Stop Band : $0.5\pi - 0.7\pi$

Magnitude in pass bands is 0dB and the suppression in the stop band is 90dB. Note that the magnitude response of the linear phase filter is given by

$$H(e^{jw}) = h_0 + 2\sum_{i=1}^{20} h_i cos(w * i) \tag{1}$$

Provide your MATLAB code for this problem:

(a) In each band, sample the frequency with $0.01\pi$ spacing so that you obtain an expression of the form

$$\mathbf{Fh} \cong \mathbf{m} \tag{2}$$

where

$$\mathbf{h} = \begin{bmatrix} h_0 & h_1 & ... & h_{20} \end{bmatrix}^T, \tag{3}$$

$$\mathbf{m} = \begin{bmatrix} m_1 & m_2 & ... & m_L \end{bmatrix}^T, \tag{4}$$

$m_i$ is the magnitude response at frequency $w_i$ and $L$ is the number of frequencies resulting from frequency domain sampling . Use standard least squares to find $\mathbf{f}$ to minimize $\| \mathbf{m} - \mathbf{Fh} \|_2^2$. Besides your MATLAB code provide plots of impulse response coefficients $h_i$ (use *stem* function of MATLAB) and a plot of the frequency response compared to the desired magnitude response (use *semilogy* function of MATLAB for this plot)

(b) Repeat part a, but now use weighted least squares as the minimization criterion where the cost function is $\| \mathbf{m} - \mathbf{Fh} \|_{\mathbf{W}}^2$ where

$$\mathbf{W} = \begin{bmatrix} m_1^{-2} & 0 & ... & 0 \\ 0 & m_2^{-2} & ... & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & m_L^{-2} \end{bmatrix} \tag{5}$$

(c) Repeat part b, but now use weighted infinity norm as the minimization criterion where the cost function is $\| \mathbf{W}_2(\mathbf{m} - \mathbf{Fh}) \|_\infty$ and

$$\mathbf{W}_2 = \begin{bmatrix} m_1^{-1} & 0 & ... & 0 \\ 0 & m_2^{-1} & ... & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & m_L^{-1} \end{bmatrix} \tag{6}$$

The MATLAB function that obtains the solution of the optimization problem

$$\text{minimize }_{\mathbf{x}} \|\mathbf{Ax} - \mathbf{b}\|_\infty$$

is given by

```
function [x,t]=mininftynorm(A,b)
% This function obtains the solution of the optimization problem
%
% minimize   || A x - b ||_\infty
%    x
%
% using linear programming.
%
%  Usage: [x,t]=mininftynorm(A,b)

% Obtain dimensions of A
[m,n]=size(A);

% Construct Linear Programming Matrices AA,bb,cc
AA=[-ones(m,1) A;
    -ones(m,1) -A];
bb=[b;-b];
cc=[1 zeros(1,n)]';
% Call linear programming function to obtain solution
z=linprog(cc,AA,bb);
% t is the minimum value
t=z(1);
% x is the solution of the optimization problem
x=z(2:length(z));
```

(d) Plot weighted error $\mathbf{W}_2(\mathbf{m} - \mathbf{Fh})$ (using plot function), for the $\mathbf{h}$ vectors obtained in part (b) and part (c) on the same figure. Comment on the magnitudes and the shape of the errors.

2. *State Evolution Exercise...*

   Consider the linear time invariant autonomous system with

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)$$

   where

$$\mathbf{A} = \begin{bmatrix} -\frac{11}{5} & \frac{4}{5} \\ \frac{6}{5} & -\frac{9}{5} \end{bmatrix}.$$

   (a) Find the expression for $\mathbf{x}(t)$ for $t \geq 0$ as a function of the initial state $\mathbf{x}(0)$.

   (b) Using MATLAB, draw $\mathbf{x}(t)$ for $t \in [0, 5]$ on the $x_1 - x_2$ plane, for $x(0) = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$. (You can take 0.01 spaced samples in time).

   (c) Repeat part b for $x(0) = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$.

   (d) Repeat part b for $x(0) = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$.

3. *Another State Evolution Exercise....*

   Repeat the previous question (parts a-c) for

$$\mathbf{A} = \begin{bmatrix} -16 & 10 \\ -25 & 14 \end{bmatrix}.$$

4. *Complete Response for an LTI System...*

   Given the state space description,

$$\mathbf{A} = \begin{bmatrix} 1/4 & 0 & 0 \\ 1 & 1/3 & -1 \\ 0 & 0 & 2 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix},$$

   and $D = 0$, Find the complete response for

$$\mathbf{X}(0) = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, \qquad \mathbf{U}(t) = e^{-t}$$

5. Find the transfer matrix $\mathbf{H}(s)$ for

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & -4 \\ 1 & 0 & 7 \\ 0 & 1 & 11 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix},$$

and $D = 0$.

6. *Matrix Exponential....*

Consider the system described by $\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t)$ where $A = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$

(a) Find $e^{\mathbf{A}}$.

(b) Suppose $x_1(0) = 1$ and $x_2(1) = 2$. Find $\mathbf{x}(2)$.