Homework Set #2

Due: Tuesday, February 25, 2014.

Please attach the MATLAB codes for the problems with the MATLAB parts.

1. *An Algebraic Exercise on Covariance*

   Let X_1 and X_2 be the components of the random vector $\mathbf{x}$ whose covariance matrix is

$$\mathbf{C}_{\mathbf{x}} = \begin{bmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{bmatrix}. \tag{1}$$

   Find the variance of the random variable $Z = 3X_1 - 2X_2$.

2. *Bayesian or non-Bayesian.... Kay 1.3*

   Let $x = \Theta + w$ where w is a random variable with PDF $p_w(w)$.

   (a) If Θ is a deterministic parameter, find the PDF of x in terms of p_w and denote it by $p(x; \Theta)$.

   (b) Now assume that Θ is a random variable independent of w and find the conditional pdf of $p(x|\Theta)$.

   (c) Finally, do not assume that Θ and w are independent and determine $p(x|\Theta)$.

   (d) What can you say about $p(x; \Theta)$ versus $p(x|\Theta)$.

3. *Conditional Expectation, Ross 3.23*

   The conditional variance of X, given the random variable Y, is defined by

$$Var(X|Y) = E((X - E(X|Y))^2|Y). \tag{2}$$

   Show that

$$Var(X) = E(Var(X|Y)) + Var(E(X|Y)) \tag{3}$$

4. *Random Sum, Ross 3.24*

   Let $\{X_k, k \in Z\}$ be a i.i.d. random process (with mean μ_x and variance σ_X^2 and let N be a random variable independent of $\{X_k\}$, with mean μ_N and variance σ_N^2. Derive the variance of

$$Z = \sum_{i=1}^{N} X_i \tag{4}$$

   in terms of known parameters.

5. *Some Linear Systems flavour*

Let X_k be a wide sense stationary process with zero mean and correlation function $R_X(\tau)$. This process is filtered by an FIR system with transfer function $H(z) = h_0 + h_1 z^{-1}$. Let the output of the system be Y_k. Find the covariance matrix of the vector

$$\mathbf{W} = \begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}. \tag{5}$$

6. *Simulating a Gaussian Vector*

(a) Write a MATLAB code to generate a 2 dimensional Gaussian vector

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

with

- $E(X_1) = 0.5,\ E(X_2) = 0.3$
- $\sigma^2_{X_1} = 1\ ,\sigma^2_{X_2} = 2,$
- $Cov(X_1, X_2) = 0.6.$

Hint: Try writing each component of X as function of two independent Gaussian random variables.

(b) Generate 10000 vectors with this code and plot them on the X_1X_2 plane as points (i.e., a scatter diagram).

(c) Based on the samples $\mathbf{X}_i$ you generated in the previous part, calculate

$$\frac{1}{N}\sum_{i=1}^{N} \mathbf{X}_i \mathbf{X}_i^T \tag{6}$$

where $N = 10000$. What is the limit of the above expression as $N \to \infty$?