ECOE530/ELEC430 Handout #7
Detection and Estimation Theory Friday April 18, 2014

**Homework Set #6**
Due: Friday April 25, 2014.

Please attach the MATLAB codes for the problems with the MATLAB parts.

1. *Exponentials in Noise .... Kay 4.1*

   We wish to esitmate the amplitudes of exponentials in noise. The observed data are

$$x[n] = \sum_{i=1}^{p} A_i r_i^n + w[n] \qquad n = 0, 1, \ldots, N-1, \tag{1}$$

   where $w[n]$ is WGN with zero mean and variance $\sigma^2$. Find the MVU estimator of the amplitudes also their covariance. Evaluate your results for the case when $p = 2$, $r_1 = 1$, $r_2 = -1$ and $N$ is even.

2. *A Relative of Normal Distribution... Kay 5.2*

   The IID oservations $\{x[n], n = 0, 1, \ldots, N-1\}$ have the Rayleigh PDF

$$p(x[n]; \sigma^2) = \begin{cases} \frac{x[n]}{\sigma^2} e^{-\frac{1}{2}\frac{x^2[n]}{\sigma^2}} & x[n] \geq 0 \\ 0 & x[n] < 0 \end{cases} \tag{2}$$

   Find a sufficient statistic for $\sigma^2$.

3. *DC in Uniform Noise vs. Gaussian Noise*

   Consider the DC in noise setup where

$$x[n] = A + w[n], \qquad n = 0, \ldots, N-1 \tag{3}$$

   where $A$ is the unknown DC signal and $w[n]$ is the noise. The noise could be Gaussian or Uniform with the same variance $\sigma^2$, where the noise variance is assumed to be known.

   (a) What would be the minimum variance unbiased estimator if the noise is known to be Gaussian. Call this estimator as $\hat{\Theta}_G$.

   (b) Now for the case when the noise is known to be uniform:

      i. Show that

$$T(\mathbf{x}) = \begin{bmatrix} M(\mathbf{x}) \\ m(\mathbf{x}) \end{bmatrix} \tag{4}$$

      is sufficient statistics where

$$M(\mathbf{x}) = \max_{n \in \{0,\ldots,N-1\}} x[n], \qquad \text{and,} \tag{5}$$

$$m(\mathbf{x}) = \min_{n \in \{0,\ldots,N-1\}} x[n]. \tag{6}$$

ii. We define an unbiassed estimator $\hat{\Theta}_U$ as a linear combination of $M(\mathbf{x})$ and $m(\mathbf{x})$. Find $\hat{\Theta}_U$.

iii. Find the variance of $\hat{\Theta}_U$.

iv. Find the variance of $\hat{\Theta}_G$ when it is used for the uniform noise case.

v. Plot the variance expressions you found in the previous parts on the same graph, as a function of $N$ when $N$ is between 1 and 20.

vi. For $N = 100$, generate $x[n]$ data for $A = 5$ and for a uniform noise with $\sigma^2 = 1/12$. Apply both estimators to this data. Repeat this for 10000 realizations and calculate the average of square error (over realizations) for both estimators. Report the values you found.

vii. Repeat the previous part when the noise is Gaussian with the same variance.