ECOE530/ELEC430 Handout #9
Detection and Estimation Theory Thursday, May 1, 2014

Homework Set #7
Due: Friday, May 8, 2014.

1. *Uniform Distribution and MLE* *Kay 7.9*

   For N i.i.d. observations from $\mathcal{U}[0, \Theta]$ PDF, find the MLE of Θ.

2. *Just another type of noise and MLE ....*

   Consider our famous DC estimation setting where observations can be written as

   $$x[n] = A + w[n] \qquad n = 0, \ldots, N-1, \tag{1}$$

   where A is the unknown DC level, $w[n]$ is the zero mean i.i.d. noise and N is odd. Then noise $w[n]$ has the Laplacian distribution, with known parameter λ, i.e.

   $$f_{w[n]}(u) = \frac{\lambda}{2} e^{-\lambda|u|} \qquad u \in \Re. \tag{2}$$

   (a) Determine the maximum likelihood estimator for A.

   (b) For $N = 10001$, in MATLAB, generate samples of $x[n]$ and estimate A based on the MLE you found in the previous part. Alternatively estimate A using sample average estimator. Repeat this experiment 1000 times and compare the average square errors of both estimators.

3. *Back to the Direction Finding Problem....*

   You are still working in MIELIND and after your succesful performance analysis of the antenna array for the direction finding problem (which was not more than one month ago), now you are assigned to design the algorithm to estimate the angle of arrival from antenna array measurements. Remember that the DF system consists of a uniform linear array with M antennas (Antenna-0, to Antenna-(M-1)) as shown in Figure 1. Antenna separation is specified with the d parameter. The angle of the electromagnetic wave is defined as β, which is the angle between the array line and the electromagnetic wave direction line. We record a single snapshot of both in phase and out of phase components of the measured signal value at each antenna which are specified by x_n^I and x_n^Q respectively for $n = 0, \ldots, M-1$. Here

   $$x_n^I = s_n^I + w_n^I \tag{3}$$
   $$x_n^Q = s_n^Q + w_n^Q \tag{4}$$

   for $n = 0, \ldots, M-1$, where

   - $s_n^I = Acos(2\pi F_o \frac{d}{c} cos(\beta)n + \phi)$,
   - $s_n^Q = Asin(2\pi F_o \frac{d}{c} cos(\beta)n + \phi)$,

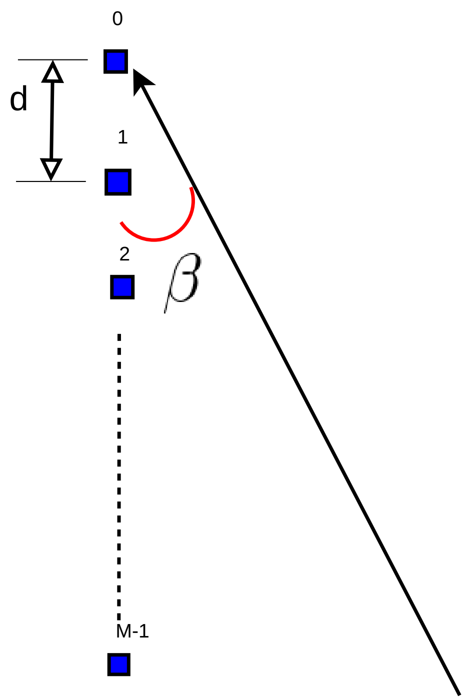

Figure 1: Direction Finding Problem Setup for Problem 4

- F_o is the frequency of the electromagnetic wave
- A is the unknown amplitude of the electromagnetic wave
- ϕ is the unknown phase (at Antenna-0)
- $w_n^I \sim \mathcal{N}(0, \sigma^2)$ is the in-phase component noise at Antenna-n,
- $w_n^Q \sim \mathcal{N}(0, \sigma^2)$ is the quadrature-phase component noise at Antenna-n,
- σ^2 is assumed to be known,
- in-phase and quadrature-phase components of the noise are independent,
- noises at different antennas are independent,
- $d = \frac{\lambda}{2}$ where $\lambda = \frac{c}{F_o}$.

In the final design, it is decided that $M = 10$. The noise standard deviation σ is equal to 0.1.

You decided to use the Maximum Likelihood estimation based on scoring method:

(a) Derive the iterative algorithm for obtaining unknown parameters $\Theta = \begin{bmatrix} A & \beta & \phi \end{bmatrix}^T$ as explicitly as you can.

(b) Now write a MATLAB code where you generate antenna array measurements corresponding to $A = 1$, $\beta = \frac{pi}{3}$ and $\phi = \frac{\pi}{5}$. Use your scoring MLE algorithm to estimate the unknown parameters. Use the following initial point $\Theta^{(0)} = \begin{bmatrix} 0.5 & \frac{pi}{2} & 0 \end{bmatrix}^T$.