Elec 206/Phys 302
Homework Assignment 1
Due on Friday, October 16, 2020, at 15:00

1 (20 pts) Give a complete proof of the identity $\sum_{i=1}^{3} \epsilon_{ijk}\epsilon_{i\ell m} = \delta_{j\ell}\delta_{km} - \delta_{jm}\delta_{k\ell}$ following the steps discussed in the Lecture on Oct. 07, 2020, i.e., write down the details of the argument reducing the proof to the study of 9 specific cases and prove the validity of the identity for these 9 cases by calculating its left- and right-hand sides.

2 (10 pts) Establish the following identity for every $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^3$.

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) = \mathbf{0}\,.$$

3 (20 pts) Calculate the gradient and Laplacian of the function, $f(\mathbf{r}) := r^n$, where $\mathbf{r} := (x, y, z)$, $r := |\,\mathbf{r}\,|$, and n is a positive of negative integer.

4 (20 pts) Calculate the divergence of the vector-valued function, $\mathbf{A}(\mathbf{r}) := \dfrac{\mathbf{r} - \mathbf{a}}{|\,\mathbf{r} - \mathbf{a}\,|^n}$, where $\mathbf{r} := (x, y, z)$ and $\mathbf{a}$ is a constant vector.

5 (30 pts) Establish the following identities for every scalar function $f(\mathbf{r})$ and vector-valued functions $\mathbf{A}(\mathbf{r})$ and $\mathbf{B}(\mathbf{r})$, where $\mathbf{r} := (x, y, z)$

5.a) $\nabla \cdot [f(\mathbf{r})\mathbf{A}(\mathbf{r})] = \nabla f(\mathbf{r}) \cdot \mathbf{A}(\mathbf{r}) + f(\mathbf{r})\nabla \cdot \mathbf{A}(\mathbf{r})$

5.b) $\nabla \times [f(\mathbf{r})\mathbf{A}(\mathbf{r})] = \nabla f(\mathbf{r}) \times \mathbf{A}(\mathbf{r}) + f(\mathbf{r})\nabla \times \mathbf{A}(\mathbf{r})$

5.c) $\nabla \cdot [\mathbf{A}(\mathbf{r}) \times \mathbf{B}(\mathbf{r})] = \mathbf{B}(\mathbf{r}) \cdot [\nabla \times \mathbf{A}(\mathbf{r})] - \mathbf{A}(\mathbf{r}) \cdot [\nabla \times \mathbf{B}(\mathbf{r})]$