

Math 103: Midterm Exam # 1

Spring 2006

- Write your name and Student ID number in the space provided below and sign.

Student's Name:	
ID Number:	
Signature:	

- You have 75 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	
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If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1. Let a be the statement: “ $\forall x \in \mathbb{R}, \exists y \in \mathbb{Z}, \forall \delta \in \mathbb{R}^+, |x - y| < \delta.$ ”

1.a) Express the negation of a only using the qualifiers \forall and \exists . (5 points)

1.b) Determine whether a is true and give a proof of your response. (10 points)

Problem 2. Prove that $(B \Rightarrow A) \Rightarrow (B \wedge C)$ is logically equivalent to $B \wedge (A \Rightarrow C)$. (20 points)

Problem 3. Prove that

$$\bigcap_{x \in \mathbb{R}^+} (-x, x) = \{0\}. \quad (20 \text{ points})$$

Problem 4. Let A and B be sets and the power set of any set S be denoted by 2^S .

4.a) Prove that $2^{A \cap B} = 2^A \cap 2^B$. (10 points)

4.b) Prove that $2^{A-B} \neq 2^A - 2^B$, where “ $-$ ” denotes the difference set. (10 points)

Problem 5. Prove the following statement by induction.

“ $\forall n \in \mathbb{Z}^+, 3$ divides $2^{2n} - 1$. ” (25 points)