Math 103: Midterm Exam # 2

Spring 2006

- Write your name and Student ID number in the space provided below and sign.

Student's Name:	
ID Number:	
Signature:	

- You have 75 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be functions defined by

$$f(x) := \frac{1}{\sqrt{x+1}}, \qquad g(x) := \sqrt{1-x^2},$$

and A denote the interval $(-2, 2)$ in $\mathbb{R}$. Determine the following objects.

Remark: You must express your response in the form of a finite or infinite interval in $\mathbb{R}$ and show how you obtain it. You need not provide a proof for your response.

1.a) Domain of f; (5 points)

1.b) Domain of g; (5 points)

1.c) Image of A under f, i.e., $f(A)$; (5 points)

1.d) Inverse image of A under g, i.e., $g^{-1}(A)$;   (10 points)

1.e) Domain of $g \circ f$.   (10 points)

Problem 2. Let A and B be nonempty sets and $f : A \to B$ be a function with domain $\mathrm{Dom}(f) = A$. Let $\sim$ be the relation on the power set 2^A of A that is defined by

$$\forall A_1, A_1 \in 2^A, \qquad A_1 \sim A_2 \;\text{ if }\; f(A_1) \subseteq f(A_2),$$

where for every subset A' of A, $f(A')$ denotes the image of A' under f.

2.a) Show that in general $\sim$ is not a partial ordering relation on 2^A. (15 points)

2.b) Find a condition on f such that $\sim$ is a partial ordering relation on 2^A. You must prove that under this condition $\sim$ is a partial ordering relation on 2^A. (20 points)

Problem 3. Construct a bijection between $\mathbb{Z} - \{0\}$ and $\mathbb{Z}^+$. You must define

$$f : \mathbb{Z} - \{0\} \to \mathbb{Z}^+$$

by giving a formula for $f(n)$ for all $n \in \mathbb{Z} - \{0\}$, and show that f is a bijection (30 points)