Math 103: Midterm Exam # 3

Spring 2006

- Write your name and Student ID number in the space provided below and sign.

Student's Name:	
ID Number:	
Signature:	

- You have 80 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1. Let A, B and C be nonempty sets and $f : A \to B$ and $g : B \to C$ be arbitrary functions. Prove that

$$\forall D \subseteq C, \qquad (g \circ f)^{-1}(D) = f^{-1}(g^{-1}(D)),$$

where for any nonempty sets X and Y, any function $h : X \to Y$, and any $Z \subseteq Y$, $h^{-1}(Z)$ denotes the inverse image of Z under h. (20 points)

Problem 2. Let X and Y be nonempty sets. Prove that $|X| \leq |X \times Y|$, where for any set Z, $|Z|$ denotes the cardinality of Z. (20 points)

Problem 3. Let A be a nonempty set and S be the set of all functions $f : A \to A$ with domain A. Prove that $|A| \leq |S|$, where for any set Z, $|Z|$ denotes the cardinality of Z. (30 points)

Problem 4. Let $\mathcal{F}$ be the set of all sequences $s : \mathbb{Z}^+ \to \{0, 1\}$ in $\{0, 1\}$.

4.a) Show that there is a sequence of distinct terms in $\mathcal{F}$. (10 points)

4.b) Prove that $\mathcal{F}$ is uncountable. (20 points)

Hint: Try to use Cantor's diagonalization argument.