

Math 103, Spring 2006, Homework Set # 1¹

Due on Feb. 28, 2006, at 12:30

- 1) Let a, b, c be statements and

$$d : a \rightarrow (b \rightarrow c), \quad e : (a \rightarrow b) \rightarrow c.$$

Compare the truth values of d and e . Is $d \Leftrightarrow e$ a true statement?

- 2) Use the truth tables to determine which of the following statements are equivalent to each other. Here p and q are statements.

$$a : (p \wedge q) \vee (\sim p \wedge \sim q),$$

$$b : \sim p \vee q,$$

$$c : (p \vee \sim q) \wedge (q \vee \sim p),$$

$$d : \sim (p \vee q),$$

$$e : (q \wedge p) \vee \sim p,$$

- 3) Use the truth tables to determine which of the following statements are tautologies (always true), which are contradictions (always false), and which are neither. Here p , q and r are statements.

$$a : (p \vee q) \wedge (\sim p \vee \sim q),$$

$$b : (p \vee q) \wedge (\sim p \wedge \sim q),$$

$$c : (p \vee q) \vee (\sim p \vee \sim q),$$

$$d : [p \wedge (q \vee \sim r)] \vee (\sim p \vee r),$$

$$e : [p \rightarrow (p \rightarrow q)] \Leftrightarrow [(p \rightarrow q) \vee r],$$

$$f : [(p \wedge \sim q) \rightarrow r] \vee [r \rightarrow (q \vee \sim p)].$$

- 4) Determine the truth value of the following statements

$$a : \forall x \in \mathbb{N}, \exists y \in \mathbb{N}, x < y,$$

$$b : \exists y \in \mathbb{N}, \forall x \in \mathbb{N}, x < y,$$

$$c : \exists x \in \mathbb{N}, \forall y \in \mathbb{N}, x < y,$$

$$d : \forall y \in \mathbb{N}, \exists x \in \mathbb{N}, x < y,$$

$$e : \exists x \in \mathbb{N} \wedge \exists y \in \mathbb{N}, x < y,$$

$$f : \forall x \in \mathbb{N} \wedge \forall y \in \mathbb{N}, x < y.$$

- 5) Prove the following statements

$$5.a) \quad \forall m, n \in \mathbb{Z}, (mn \text{ is odd}) \rightarrow (m \text{ is odd}) \vee (n \text{ is odd});$$

$$5.b) \quad \forall x \in \mathbb{R}, x \in (0, 1) \rightarrow x > x^2.$$

¹Each problem will be graded out of 10 points.