

Math 103 Homework Set # 5

Due on April 25, 2006, at 12:30

I) In the following X and Y are nonempty sets, $A, B \subseteq X$, $f : X \rightarrow Y$ is a function, and $g : X \rightarrow Y$ is a one-to-one function.

- 1) Prove that $f(A \cup B) = f(A) \cup f(B)$. (10 points)
- 2) Prove that $f(A \cap B) \subseteq f(A) \cap f(B)$. (10 points)
- 3) Give an example of a function f for which $f(A \cap B) \neq f(A) \cap f(B)$. (10 points)
- 4) Prove that $g(A \cap B) = g(A) \cap g(B)$. (10 points)
- 5) Prove that $g(X - A) = g(X) - g(A)$. (10 points)
- 6) Prove that $g^{-1}(g(A)) = A$. (10 points)

DO NOT CONFUSE the inverse of a function and the inverse image of a subset under a function. The nature of the argument of a symbol will determine if it stands for the inverse of a function or the inverse image of a subset under a function.

II) Construct a bijection $f : A \rightarrow B$ for the following choices of the sets A and B . In each case prove that f is a bijection.

- 7) $A = \{2n \mid n \in \mathbb{Z}\}$ and $B = \{3m \mid m \in \mathbb{Z}^+\}$. (20 points)
- 8) $A = (0, 1)$ and $B = (a, b)$ where a and b are two arbitrary and distinct real numbers. (20 points)