Math 103 Homework Set # 6

Due on May 09, 2006, at 12:30

1) Let $n \in \mathbb{Z}^+$ and for every $i \in \{1, 2, \cdots, n\}$, A_i be a countably infinite set. Prove that $\bigcup_{i=1}^{n} A_i$ is countably infinite. (20 points)

2) Let $\{A_i\}_{i \in \mathbb{Z}^+}$ be a family of countably infinite sets A_i. Is $\bigcup_{i \in \mathbb{Z}^+} A_i$ countably infinite? Prove your response. (20 points)

3) Let $f : A \to B$ be a one-to-one function with $\mathrm{Dom}(f) = A$. Show that if B is countable, then so is A. (20 points)

4) Let $f : A \to B$ be an onto function with $\mathrm{Dom}(f) = A$. Show that if A is countable, then so is B. (20 points)

5) Prove that if an uncountable set is partitioned into a finite number of its subsets then at least one of these subsets must be uncountable. (20 points)

6) Let A be an uncountable set, τ be a countable index set, and $\{A_\alpha\}_{\alpha \in \tau}$ be a partition of A. Does this imply that at least one of A_α's must be uncountable? Why? (20 points)