Math 103 Homework Set # 7

1) Prove that the power sets of two equivalent sets are equivalent, i.e., for any pair of sets A and B, $A \sim B$ implies $2^A \sim 2^B$.   (30 points)

2) Let A and B be disjoint sets, i.e., $A \cap B = \emptyset$. Show that $2^A \times 2^B$ is equivalent to $2^{A \cup B}$.   (30 points)

3) Let $A = \mathbb{N} = \{0, 1, 2, 3, \cdots\}$ and $B = \mathbb{Z}^- = \{-1, -2, -3, \cdots\}$. Use the statements of Problems 1 and 2 and the fact that $A \cap B = \emptyset$, $A \cup B = \mathbb{Z}$, $A \sim B \sim \mathbb{Z}$ and $\mathbb{R} \sim 2^{\mathbb{Z}}$ to prove that $\mathbb{R}^2 := \mathbb{R} \times \mathbb{R}$ is equivalent to $\mathbb{R}$.   (20 points)

4) Prove that for all $n \in \mathbb{Z}^+$, $\mathbb{R}^n$ is equivalent to $\mathbb{R}$..   (20 points)