2. Give the statement of   See the book.

2.a) well-ordering Axiom: (5 points)

2.a) well-ordering Principle: (5 points)

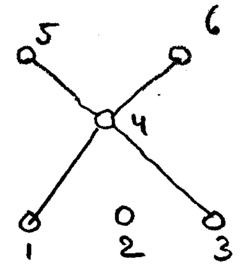
3. Let $(A, \preceq)$ be a poset and $C \subseteq A$. Prove that if it exists the infimum of C in A is unique. (20 points)   See the book (Thm 5.4.2).

4. Let $A = \{1, 2, 3, 4, 5, 6\}$ find a partial ordering R on A such that the poset (A, R) has three minimal and three maximal elements. Express your result as a subset of A^2. (20 points)

Min. elements : 1, 2, 3

Max. elements : 2, 5, 6

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6),$
$(1,4), (3,4), (1,6), (1,5), (3,5), (3,6),$
$(4,5), (4,6)\}$



5. Prove that $(\mathcal{P}(\mathbb{N}) \setminus \{\mathbb{N}\}, \subseteq)$ does not have a greatest element (30 points)

By ※ Suppose that $(\mathcal{P}(\mathbb{N}) \setminus \{\mathbb{N}\}, \subseteq)$ has a greatest element, say A.

$A \in \mathcal{P}(\mathbb{N}) \setminus \{\mathbb{N}\} \Rightarrow A \subsetneq \mathbb{N} \Rightarrow \exists n \in \mathbb{N},\ n \notin A.$

$\Rightarrow \{n\} \subseteq \mathbb{N}$ and $\{n\} \neq \mathbb{N} \Rightarrow \{n\} \subsetneq \mathbb{N}$

So $\{n\} \in \mathcal{P}(\mathbb{N}) \setminus \{\mathbb{N}\}$ $\Longrightarrow$ A is not the greatest element of $\mathcal{P}(\mathbb{N}) \setminus \{\mathbb{N}\}$ ※.

But because $n \notin A$, $\{n\} \not\subseteq A$

So by ※ $\mathcal{P}(\mathbb{N}) \setminus \{\mathbb{N}\}$ has no greatest element. ∎