Solutions.

Math 103: Quiz # 8

Fall 2007

- Write your name and Student ID number in the space provided below and sign.

Name, Last Name:	
ID Number:	
Signature:	

- You have 50 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the quiz within the first 5 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)

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1. Give the definition of the following terms. See the book.

1.a) a sequence in a set: (5 points)

1.b) a convergent real sequence: (5 points)

1.c) a real series: (5 points)

1.d) a strictly decreasing sequence in a poset: (5 points)

1.e) equivalent sets: (5 points)

2. Let $s : \mathbb{Z}^+ \to \mathbb{R}$ be a real sequence. Prove that if the series $\sum_{n=1}^{\infty} s_n$ converges, then $s_n \to 0$. (25 points) See thm 6-6-1 of the book.

3. Let $s, t : \mathbb{Z}^+ \to \mathbb{R}$ be convergent real sequences and $u : \mathbb{Z}^+ \to \mathbb{R}$ be the real sequence defined by: $\forall n \in \mathbb{Z}^+$, $u_n := s_n + t_n$. Prove that if $s_n \to 1$ and $t_n \to -1$, then (u_n) converges to zero. (25 points) (25 points)

$s_n \to 1 \Rightarrow \forall \epsilon_1 \in \mathbb{R}^+, \exists N_1 \in \mathbb{Z}^+, \forall n \in \mathbb{Z}^+, (n \geq N_1) \Rightarrow |s_n - 1| < \epsilon_1$

$t_n \to -1 \Rightarrow \forall \epsilon_2 \in \mathbb{R}^+, \exists N_2 \in \mathbb{Z}^+, \forall m \in \mathbb{Z}^+, (m \geq N_2) \Rightarrow |t_m + 1| < \frac{\epsilon}{2}$

$\forall \epsilon \in \mathbb{R}^+$, let $\epsilon_1 := \frac{\epsilon}{2}$, $\epsilon_2 := \frac{\epsilon}{2}$, and $N := N_1 + N_2 \in \mathbb{Z}^+$

Then $\forall p \in \mathbb{Z}^+$, $p \geq N \Rightarrow (p \geq N_1) \wedge (p \geq N_2)$

$\Rightarrow |s_p - 1| < \epsilon_1 \wedge |t_p + 1| < \epsilon_2$

$\Rightarrow |u_p| = |s_p + t_p| = |s_p - 1 + t_p + 1| \leq |s_p - 1| + |t_p + 1|$

$< \epsilon_1 + \epsilon_2 = \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$

This proves $u_n \to 0$.

4. Let A, B, C be sets. Show that in general $A \sim B$ does not imply $A \cup C \sim B \cup C$ (25 points)

Let $A := \{1\}$, $B := \{2\}$ and $C := \{2\}$

clearly $A \cup C = \{1, 2\} = I_2$, $B \cup C = \{2\}$

so $B \cup C \subsetneq I_2 \Rightarrow$ there is no onto function $f : B \cup C \to I_2 \Rightarrow$ there is no bijection $f : B \cup C \to I_2 = A \cup C \Rightarrow A \cup C$ is not equivalent to $B \cup C$.

But $\varphi : A \to B$, defined by $\varphi(1) := 2$ is clearly a bijection $\Rightarrow A \sim B$.

This shows that $(A \sim B) \Rightarrow (A \cup C \sim B \cup C)$ is false in general.