

Name, Last Name:

Grade:

Student No:

## Math 103: Quiz # 4

Spring 2007

You have 45 minutes.

1. Let  $A, B, C$  be sets,  $D \subseteq B$ ,  $S$  be a relation that relates elements of  $A$  to those of  $B$ , and  $T$  be a relation that relates the elements of  $B$  to those of  $C$ . Give the definition of

1.a) the identity relation associated with  $A$ ; (5 points)

1.b) the range of  $S$ ; (5 points)

1.c) the inverse image of  $D$  under  $S$ ; (5 points)

1.d) the composite relation  $T \circ S$ ; (5 points)

1.e) the inverse relation to  $S$ . (5 points)

2. Let  $R := \{(x, y) \in \mathbb{R}^2 \mid 1 < x < y\}$  and  $I := (0, 2) := \{r \in \mathbb{R} \mid 0 < r < 2\}$ .

2.a) Find the domain of  $R$ ; (10 points)

2.b) Prove that  $\text{Ran}(R) = (1, \infty) := \{r \in \mathbb{R} \mid r > 1\}$ ; (10 points)

2.c) Find the inverse image of  $I$  under  $R$ . (10 points)

3. Let  $A$  and  $B$  be sets,  $D \subseteq B$ , and  $S \subseteq A \times B$ .

3.a) Prove that if  $\text{Ran}(S) \subseteq D$ , then  $S^{-1}(\text{Ran}(S)) = \text{Dom}(S)$ . (10 points)

3.b) Prove that  $\text{Dom}(S^{-1} \circ S) = \text{Dom}(S)$ , where  $S^{-1}$  denotes the inverse relation to  $S$ .  
(15 points)

4. Let  $R := \{(x, y) \in \mathbb{R}^2 \mid x = \sqrt{y+1}\}$  and  $S := \{(x, y) \in \mathbb{R}^2 \mid x = \frac{1}{\sqrt{|y|}}\}$ . Find the domain of  $S \circ R$  and give an argument to support your response. (20 points)