Name, Last Name: **Grade:**

Student No:

Math 103: Quiz # 7

Spring 2007

<u>You have 40 minutes.</u>

1. Give the definition of the following terms.

1a) A transposition of $I_n := \{1, 2, \cdots, n\}$ where $n \in \mathbb{Z}^+$. (5 points)

1b) A permutation of $I_n := \{1, 2, \cdots, n\}$ where $n \in \mathbb{Z}^+$. (5 points)

2. Let $\sigma : I_4 \to I_4$ be the permutation defined by

$$\sigma := \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix}.$$

i.e., $\sigma(1) := 4$, $\sigma(2) := 2$, $\sigma(3) := 1$, $\sigma(4) := 3$. Express σ as the composition of a pair of transpositions of I_4, i.e., find transpositions θ_1 and θ_2 such that $\sigma = \theta_2 \circ \theta_1$. (15 points)

3. Let A and B be sets, $f : A \to B$ be a function, and $D := \mathrm{Dom}(f)$. Prove that if f is one-to-one, $f^{-1} \circ f = \mathrm{Id}_D$. (25 points)

4. Let A, B, C be nonempty sets, $C \subseteq B$, and $f : A \to B$ be an invertible function. Prove that the inverse image of C under f is equal to the image of C under the inverse function f^{-1} of f. (25 points)