Solutions

Math 207: Quiz # 1A

Fall 2004

- You have 35 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the quiz within the first 5 minutes. After this time for any question you may want to ask 2 points will be deduced from your grade (You may or may not get an answer to your question(s).)

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1. Determine which of the following series converges. Explain your reasoning.

1.a)

$$\sum_{n=0}^{\infty} \frac{1}{(n+2)\ln(n+2)} \qquad \text{(3 points)}$$

Hint: Use the integral test.

let $f(x) = \dfrac{1}{(x+2)\ln(x+2)} \Rightarrow \displaystyle\int_0^{\infty} f(x)\,dx = \int_0^{\infty} \frac{dx}{(x+2)\ln(x+2)}$

let $u = \ln(x+2) \Rightarrow du = \dfrac{dx}{x+2} \Rightarrow$

$$\int \frac{dx}{(x+2)\ln(x+2)} = \int \frac{du}{u} = \ln|u| + c$$

$$\Rightarrow \int_0^{\infty} \frac{dx}{(x+2)\ln(x+2)} = \ln|\ln(x+2)|\Big|_0^{\infty} = \infty \text{ : diverges}$$

So the series <u>diverges</u>.

1.b)

$$\sum_{n=0}^{\infty} \underbrace{\frac{(-1)^n}{\ln(n+2)}}_{a_n} \qquad \text{(2 points)}$$

This is an alternating series for $\forall n$ $|a_n| > |a_{n+1}|$

and $a_n \to 0$ so it <u>converges</u>.

2. Use the power series expansion of e^{-t^2} about $t = 0$ to estimate the value of

$$f(x) = \int_0^x e^{-t^2} dt$$

at $x = 0.1$. (7 points)

$$e^{-t^2} = 1 + (-t^2) + \frac{1}{2!}(-t^2)^2 + \frac{1}{3!}(-t^2)^3 + \dots$$

$$= 1 - t^2 + \frac{t^4}{2!} - \frac{t^6}{3!} + \dots$$

$$\Rightarrow f(x) = \int_0^x e^{-t^2} dt = t - \frac{t^3}{3} + \frac{t^5}{2 \times 5} - \frac{t^7}{6 \times 7} + \dots \Big|_0^x$$

$$= x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \dots$$

$$f(0.1) \sim 0.1 - \frac{(0.1)^3}{3} + \frac{(0.1)^5}{10} + \dots$$

$$\sim 0.1 - \frac{1}{3000} + \frac{1}{10^6} + \dots$$

$$\frac{1}{3000} \simeq 0.000333$$

So $f(0.1) \sim 0.100000 - 0.000333 + 0.000001$

$$\sim 0.099668$$

3. Let $u = 1 + \sqrt{3}i$ and $w = \sqrt{3} + i$. Find the modulus $|z|$, the principal argument $\arg_P(z)$, the real part $\mathrm{Re}(z)$, and the imaginary part $\mathrm{Im}(z)$ of

$$z = \frac{u^2}{\bar{w}^3}. \qquad \text{(8 points)}$$

Hint: First determine the polar form of u and w.

$|u| = \sqrt{1+3} = 2, \qquad |w| = \sqrt{3+1} = 2$

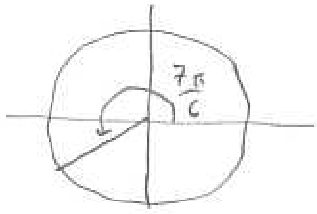
$\arg_P(u) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}, \qquad \arg_P(w) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

$u = 2e^{\frac{i\pi}{3}}, \qquad w = 2e^{\frac{i\pi}{6}} \Rightarrow \bar{w} = 2e^{-\frac{i\pi}{6}}$

$u^2 = 4e^{\frac{2i\pi}{3}}, \qquad \bar{w}^3 = 8e^{-\frac{i\pi}{2}}$

$$\Rightarrow z = \frac{4e^{\frac{2i\pi}{3}}}{8e^{-\frac{i\pi}{2}}} = \frac{1}{2}e^{i\pi\left(\frac{2}{3}+\frac{1}{2}\right)}$$

$$\Rightarrow z = \frac{1}{2}e^{\frac{7i\pi}{6}} = \frac{1}{2}\left[\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right]$$



$|z| = \frac{1}{2} \qquad \arg_P(z) = \frac{7\pi}{6}$

$\mathrm{Re}(z) = \frac{1}{2}\cos\left(\frac{7\pi}{6}\right) = -\frac{1}{2}\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{4}$

$\mathrm{Im}(z) = \frac{1}{2}\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}\sin\left(\frac{\pi}{6}\right) = -\frac{1}{4}$