Solution

Name: Student ID: Signature:

Math 207: Quiz # 4A

Fall 2004

- You have 25 minutes.
- You may ask any question about the quiz within the first 5 minutes. After this time for any question you may want to ask 2 points will be deduced from your grade.

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1. Let $f(z) = \dfrac{e^{(z+2)^2}}{z^2+4z+5}$ and $C := \{z \in \mathbb{C} \mid |z| = 3\}$.

1.a) Find the poles of f and determine their order. (8 points)

$z^2+4z+5 = (z+2)^2+1 = 0 \Rightarrow z = -2 \pm i =: z_\pm$ are the poles

$$\lim_{z\to z_-}(z-z_-)f(z) = \lim_{z\to z_-}\frac{e^{(z+2)^2}}{(z-z_+)} = \frac{e^{(-2-i+2)^2}}{-2-i-(-2+i)}$$

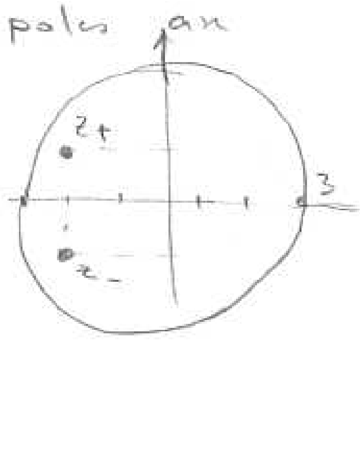
$$= \frac{e^{-1}}{-2i} = \frac{i}{2e} \text{ is finite \& nonzero} \Rightarrow R(z_-) = \frac{i}{2e}$$

$$\lim_{z\to z_+}(z-z_+)f(z) = \lim_{z\to z_+}\frac{e^{(z+2)^2}}{(z-z_-)} = \frac{e^{(-2+i+2)^2}}{-2+i-(-2-i)} = \frac{e^{-1}}{2i}$$

$\Rightarrow R(z_+) = \dfrac{1}{2ie}$ Both poles are simple.

1.b) Calculate $\oint_C f(z)\,dz$. (5 points)

$|z_\pm| = |-2\pm i| = \sqrt{5} < 3$ so both poles are inside C $\Rightarrow$

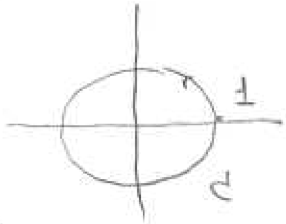


$$\oint_C f(z)\,dz = 2\pi i\left(R(z_-) + R(z_+)\right)$$

$$= 2\pi i\left(\frac{i}{2e} + \frac{1}{2ie}\right) = 0$$

2. Use the method of contour integration to calculate:

$$I := \int_0^{2\pi} \frac{d\theta}{e^{-i\theta} - 5}. \qquad \text{(7 points)}$$



let $z = e^{i\theta}$ $\quad dz = i e^{i\theta} d\theta \quad \Rightarrow \quad d\theta = \frac{dz}{iz}$

$$I = \oint_c \frac{dz}{iz\left(\frac{1}{z} - 5\right)} = \oint_c \frac{dz}{i(1 - 5z)}$$

$z = \frac{1}{5}$ is a pole and it is inside c.

$$\lim_{z \to \frac{1}{5}} \left(z - \frac{1}{5}\right) \cdot \frac{1}{i(1 - 5z)} = \frac{1}{i(-5)} = -\frac{1}{5i}$$ is finite & non zero

$\Rightarrow$ Pole is simple & $R\left(\frac{1}{5}\right) = -\frac{1}{5i}$

$$\Rightarrow \quad I = 2\pi i \, R\left(\frac{1}{5}\right) = -\frac{2\pi}{5}.$$