Math 208, Spring 2013, Quiz # 1

You have 30 minutes.

Name, Last Name	Student ID Number	Signature

Problem 1 (10 points) Give the definition of the following terms.

1.a) Positivity axiom:

1.b) Infimum of a subset of $\mathbb{R}$: Let $S \subset \mathbb{R}$. Then $a \in \mathbb{R}$ is said to be the infimum of S if

i) a is a lower bound for S,

ii) If b is also a lower bound, then $a \geq b$.

Note: Greatest of lower bounds does not make sense since S may not have a lower bound!!! If you want to define this way, you should first assume that S is bounded from below.

1.c) Completeness axiom:

1.d) Inductive set:

1.e) Set of natural numbers:

Problem 2 (10 points) Prove that the product of every two natural numbers is also a natural number.

Let $n \in \mathbb{N}$ and define

$$S(n) := \{ n \cdot m \in \mathbb{N} \text{ for all } m \in \mathbb{N} \}$$

For $n = 1$, $1 \cdot m = m \in \mathbb{N}$ for $m \in \mathbb{N}$.

Assume that $S(n)$ is true for some $n \in \mathbb{N}$.

Then for $m \in \mathbb{N}$

$$m(n+1) = mn + m$$

$S(n)$ implies that $mn \in \mathbb{N}$. Since $m \in \mathbb{N}$ and sum of two natural numbers is natural we have $m(n+1) \in \mathbb{N}$.

Hence $S(n) \Rightarrow S(n+1)$ and this completes the induction.

Note; $S(n) := \{ n \cdot m \in \mathbb{N}$ for all $n \in \mathbb{N}$ and $m \in \mathbb{N} \}$

This is not well-defined.

Problem 3 (10 points) Let $\forall a \in \mathbb{R}$, $S_a := \{x \in \mathbb{Q} \,|x < a\}$. Show that $\sup(S_a) = a$.

Let $a \in \mathbb{R}$. By definition of S_a a is an upper bound and $\sup(S_a)$ exist. By definition of $\sup(S_a)$, $\sup(S_a) \leq a$. Assume that $\sup(S_a) < a$. Since $\mathbb{Q}$ is dense in $\mathbb{R}$ $\exists\, s \in (\sup(S_a), a) \cap \mathbb{Q}$. Since $s \in \mathbb{Q}$ and $s < a$, $s \in S_a$. But $s > \sup(S_a)$ which is a contradiction. Hence $\sup(S_a) = a$.