Math 303, Fall 2006
Assignment for Dec. 14-18

- Read pages 824-839 and 845-855 of the textbook (Riley-Hobson-Bence, 3rd Edition).
- Solve Problems 24.1, 24.3, 24.4 of the textbook.
- Solve the following problems.

1. Find the real and imaginary parts $u(x,y)$ and $v(x,y)$ of the following functions and determine if there is a region in $\mathbb{C}$ where they are analytic.

$$\begin{aligned} f(z) &= \frac{2z+3}{z^2+2}, \\ g(z) &= \frac{\sinh(z)}{z^*}, \end{aligned}$$

2. Let $f:\mathbb{C}\to\mathbb{C}$ and $g\mathbb{C}\to\mathbb{C}$ be analytic functions in a region $D\subseteq\mathbb{C}$. Prove that their product is also analytic in this domain and that $[f(z)g(z)]' = f'(z)g(z)+f(z)g'(z)$.
3. Find the analogues of the Cauchy-Riemann conditions in the polar coordinates (r,θ), i.e., let $u(r,\theta)=\Re[f(r\,e^{i\theta})]$ and $v(r,\theta)=\Im[f(r\,e^{i\theta})]$, where $\Re$ and $\Im$ respectively stand for the real and imaginary parts of their argument, and find conditions on u and v such that f is analytic at some $z\in\mathbb{C}$.
4. Show that $u(x,y)=\cosh y\cos x$ satisfies the Laplace's equation. Find an analytic function $f(z)$ such that $u(x,y)$ is the real part of $f(x+iy)$.
5. Repeat the previous problem for $u(x,y)=\dfrac{y}{(1-x)^2+y^2}$.
6. Evaluate the following contour integral along the circle (C) defined by $|z|=3$.

$$\oint_C \frac{e^{3z}}{z-\ln 2}\,dz,$$
$$\oint_C \frac{e^{3z}}{(z-\ln 2)^2}\,dz,$$
$$\oint_C \frac{e^{3z}}{(z-\ln 2)^3}\,dz.$$