Math 303: Midterm Exam # 1

Fall 2006

- Write your name and Student ID number in the space provided below and sign.

Student's Name:	
ID Number:	
Signature:	

- You have 80 minutes.
- You may use any statement which has been proven in class, except for the cases where you are asked to reproduce the proof of that statement.
- You may ask any question about the exam within the first 10 minutes. After this time for any question you may want to ask 5 points will be deduced from your grade (You may or may not get an answer to your question(s).)
- (Optional) Grade your own work out of 100. Record your estimated grade here:

Estimated Grade:	

If your expected grade and actual grade will turn out to differ by 9 points or less, you will be given the highest of the two.

To be filled by the grader:

Actual Grade:	
Adjusted Grade:	

Problem 1. Let $a := i^{\sqrt{2}}$ and $b := (\sqrt{2})^i$. Find the polar representation of all values of the product of a and b. (15 points)

Problem 2. Let $F : \mathbb{R}^2 \to \mathbb{R}$ be defined by $F(x, y) := \frac{y^2}{2} - e^x(y - 1)$ for all $x, y \in \mathbb{R}$. Find the stationary point(s) of F and determine whether it is (they are) a maximum, minimum, or saddle point. (20 points)

Problem 3. Let $\vec{A} : \mathbb{R}^3 \to \mathbb{R}^3$ and $\vec{B} : \mathbb{R}^3 \to \mathbb{R}^3$ be differentiable functions and $\vec{C} := \vec{\nabla} \times (\vec{A} \times \vec{B})$. Use the properties of the Kronecker delta and Levi Civita epsilon symbols to show that the component of C along the x-axis is given by

$$\vec{B} \cdot \vec{\nabla} A_1 - \vec{A} \cdot \vec{\nabla} B_1 + A_1 \vec{\nabla} \cdot \vec{B} - B_1 \vec{\nabla} \cdot \vec{A},$$

where A_1 and B_1 are respectively components of $\vec{A}$ and $\vec{B}$ along the x-axis. (15 points)

Problem 4. Give the statement of the Stokes theorem and prove it (using the Green's theorem in the plane) for a vector field $\vec{A}$ that is defined on the following surface. (25 points)

$$S := \{(x, y, z) \in \mathbb{R}^3 \mid x = 0,\ |y| \leq 1,\ |z| \leq 2\ \}.$$

Problem 5. Let S be the boundary of the region V defined by

$$V := \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 \leq 1,\ 0 \leq z \leq 1\ \},$$

$\hat{n}$ be the unit outward normal vector to S, and $\vec{J} : \mathbb{R}^3 \to \mathbb{R}^3$ be the vector field defined by:

$$\vec{J}(x, y, z) = yz\,\hat{\imath} - xz^2\,\hat{\jmath} + (x^2 + y^2)z^3\,\hat{k},$$

where $\hat{\imath}, \hat{\jmath}, \hat{k}$ are the unit vectors along positive x-, y-, and z-axes. Compute the surface integral of $\vec{J} \cdot \hat{n}$ on S, i.e., $\iint_S \vec{J} \cdot \hat{n}\, ds.$ (25 points)

Hint: Use divergence theorem.